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PREFACE

“God created the integers, rest all is work of man.”

Mathematics is embedded in the structure of the Universe. Although mathematical systems are free inventions of human minds, they have astonishing applications to nature. This prompted Eugene Wigner, the great mathematician to call this the “unreasonable effectiveness of mathematics”.

Many areas of mathematics begin with an analysis of the real world problems. Algebra was originated for solving problems in arithmetic. Geometry has its origin in calculation of distances, angles and areas. Statistics has its origin in calculation of probabilities in gambling. Abstraction is the process of generalizing the essence of a mathematical concept, removing all dependence on the real world problem.

The pedagogical attitude towards the subject turns away then the not so ambitious student from pursuing Mathematics. It is pursued from a structural or application point of view. Tools are built up in a logical, sequential sequence, smaller pieces fitting accurately to larger blocks. This is the main reason for most of students to think after 12 years of schooling that mathematics is a pointless exercise, and its application to real life is no more than totaling the grocery’s bill.

Amateurs have “attacked” the foundation, found proofs for century old problems, calculated larger primes, and found trillionth digit of pi and many more amazing things. Years ago many of the discoveries were made by military persons, officers, and lawyers. Actually pure mathematics was not a bread earner just a century ago. The amateurs cannot surpass the brilliant mathematicians, however exciting explorations are possible.

The author has covered topics which may have arisen in the mind of all those who have had years of mathematics teaching, and answers to their queries are generally beyond the scope of syllabus of examination and thus of teacher’s knowledge. The mediocre teacher will never want students to ask much and the pedagogue will kill the curiosity at the first instance.

This book has attempted to provide a set of off beat, fun filled pages. Most of the mothers had a tough time explaining to their child that 11 is eleven and not oneteen and 12 is twelve and not twoteen. The early learner would have often wondered why of all bases this strange number 2.718... was chosen as natural logarithm base called, e . After all why is Mathematics left out of the Nobel Prize by the Academy? Did you know that only a few folds to a paper could take you to the moon? Binary system is okay but why the computer uses such a strange notation as AD45F3? A few brilliant logical and dissection puzzling stories with solutions are to be found here.

The reader would find unbelievable things like there exists a triangle in the universe whose angles add upto more than 180° .

What made 1729 the taxi number so famous? Some strange path can make a square tyred bicycle move smoothly. All this and more interesting stories are to be found in the book. This book is not organized like a textbook, it is collection of several thoughts, articles, brilliant number sequences and puzzles. These have been collected over years of “adventuring” in mathematics, with only love and passion. It is not for serious mathematicians, not for people who know too much and find the book no more than a carbon copy of their own reproductions. The author claims absolutely no originality in the work. The book should be read with a rough page and pencil, that’s the only way to enjoy it.

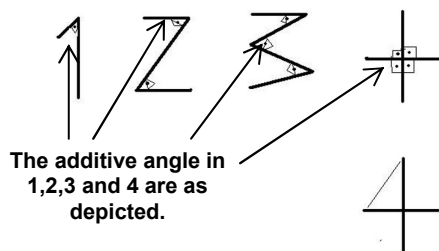
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Chapter 1 EVOLUTION OF NUMBERS

Most of us may have wondered on the origin of the symbols for all numbers. How a particular notation came into existence, and why some particular notations found universal acceptance are exciting questions. All Arabic numbers we use today as international numerals, are ideograms created by Abu Ja'far Muhammad ibn Musa *al-Khowarizmi* (c.778 - c.850).

It is opined that he used the abacus notation for developing the manuscript for decimal system. Incidentally, for those who are unaware ABACUS is a calculating device, probably of Babylonian origin, that was long important in commerce. It is the ancestor of the modern calculating machine and computer. It is generally a board marked with lines and equipped with counters whose positions indicated numerical values—i.e., ones, tens, hundreds, and so on.

The numbers 1,2,3,4 were defined using additive angles.



Roots of the 1,2,3 and 4 digits

The number 1 has one angle.

The number 2 has two additives angles.

The number 3 has three additives angles.

The number 4 has four additive angles.

Probably due to cursive handwriting the number 4 gets closed.

Roots of the 5 to 10 numbers

The circle represented the hand which has five fingers. The number 5 was written under the line. Number 10 placed above the line, it meant the number on top acquired double value.

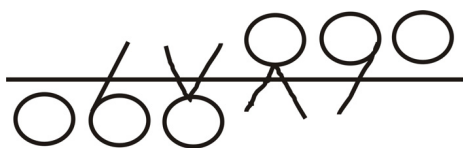
The circles, the up traces, the additive angles and the write line

To the circle five, one trace up was added, with one additive angle making the number six. To the circle five were added two up traces, with two additive angles making the number seven.

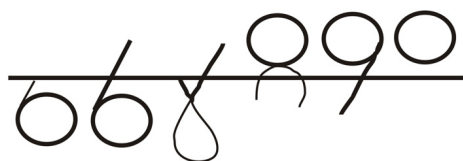
The circles, the down traces, the diminutive angles and the write line

To the circle ten was added one down trace, with one diminutive angle making the number nine.

To the circle ten were added two down traces, with two diminutives angles making the number eight.

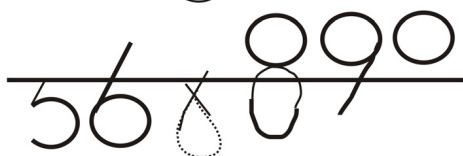


Circle below
the line represents
Hand with
5 closed fingers



so,
6 is 1 finger raised

7 is 2 fingers raised



Now circle above the line
acquires double value
10

8 is 2 fingers
less than 10

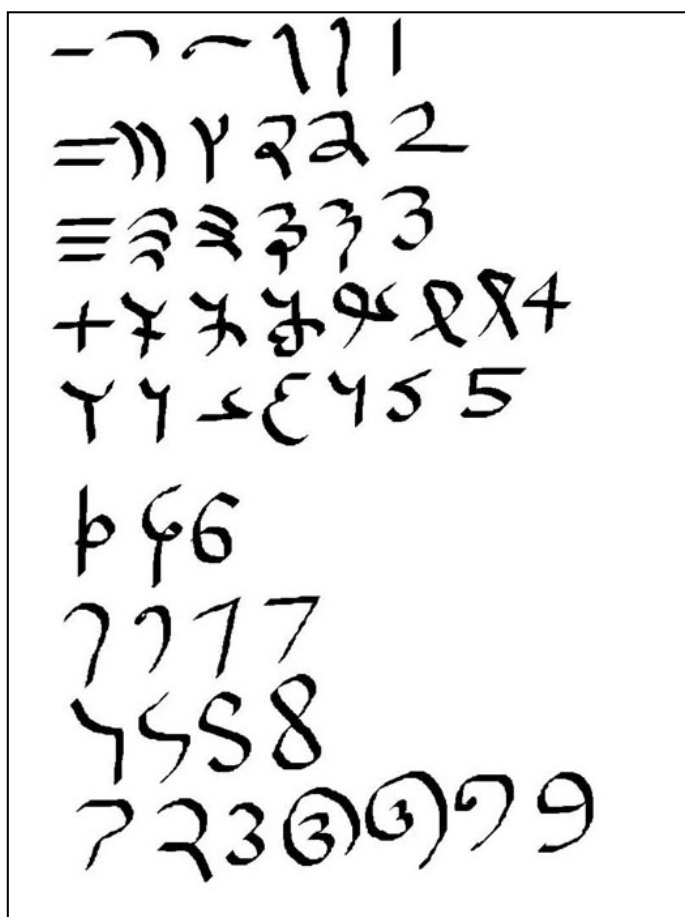
9 is 1 finger
less than 10



Gradually cursive writing
led to present symbols

The cursive handwriting makes changes on the numbers format and aesthetic. The cursive numbers five and the number seven still uses the write line on its structures.

The number seven was placed totally under the write line, and was the most simplified during its cursive evolution. First the number seven was placed under the write line. The involution of the number seven was necessary due to the similarities that the cursive seven has with the numbers.



The above figure depicts a probable evolution of symbols for the numbers. It is interesting to find that across civilizations same types of symbols evolved, some were retained in their original form, but the Arabic system found universal acceptance and is now the International Numeral system.

Chapter 2 ORIGIN OF MATHEMATICAL TERMS AND SYMBOLS

School teachers break their heads in explaining the significance of $+\times\div=\langle\rangle$ to the child. In the beginning it is all so confusing and gradually they are ingrained in the memory, and all the arithmetic is quietly done. The curious mind's journey does not end here, why $=$ for equality and a lot many similar questions. The evolution of few most used symbols and terms were researched and some very interesting stories, ideas were established. The most important written source is the definitive *A History of Mathematical Notations* by Florian Cajori.

Symbol for equality

The equal symbol ($=$) was not really in print until early 17th century. It was previously abbreviated as *aeq*. It is contended by Cajori (CAJORI, FLORIAN "A History of Mathematics", The Macmillan Company 1926) that the symbol $=$ was developed at Bologna. Robert Recorde first used the symbol in 1557 in *The Whetstone of Witte*, (1556), by Robert Recorde, a treatise on algebra. But why $=$ in particular could not be established.

Symbols for plus and minus

The introduction of the $+$ and $-$ symbols seems to be due to the Germans. The arithmetic of John Widmann, brought out in 1489 in Leipzig, is the earliest printed book in which the $+$ and $-$ symbols have been found. The $+$ sign is not restricted by him to ordinary addition; it has the more

general meaning "et" or "and" as in the heading, "regula augmenti + decrementi." The - sign is used to indicate subtraction, but not regularly so. The word "plus" does not occur in Widmann's text; the word "minus" is used only two or three times. In 1521, the symbols + and – have been used for addition and subtraction by Heinrich Schreiber, a teacher at the University of Vienna, in the arithmetic of Grammateus,. Thus, by slow degrees, the adoption of the + and - symbols became universal. Several independent paleographic studies of Latin manuscripts of the fourteenth and fifteenth centuries make it almost certain that the + sign comes from the Latin *et*. As per Cajori, the origin of the sign - is still uncertain.

The first one to make use of these signs in writing an algebraic expression was the Dutch mathematician Vander Hoecke. These symbols seem to have been employed for the first time in arithmetic, to indicate operations, by Georg Walckl in 1536. Most of the English writers of this period reserved the + and - signs as symbols of operation for algebra. Robert Recorde used it in his 1557 book, *The Whetstone of Witte*.

There seems little doubt that the sign is merely a ligature for "et", much in the same way that we have the ligature "&" for the word "and". It may have emanated from the habit of early scribes of using it as a shorthand equivalent of "m."

Symbol for division

The Anglo-American symbol for division is of 17th century origin, and has long been used on the continent of Europe

to indicate subtraction. Like most elementary combinations of lines and points, the symbol is old. It was used as early as the 10th century for the word *est*. When written after the letter "i", it symbolized "id est." When written after the word "it", it symbolized "interest." It is possible that it denoted division when written after the word "*divisa*", for "*divisa est*". There is also some evidence that some Italian algebraists used it to indicate division. In a manuscript entitled *Arithmetica and Practtica* by *Giacomo Filippo Bodi dal Aucisco*, 1684, this symbol stands for division.

The symbol " $\div$ " is called an *obelus*, and was first used for a division symbol around 1650.

Symbol for multiplication

William Oughtred (1574-1660) contributed vastly to the propagation of mathematical knowledge in English by his treatises, *The Clavis Mathematicae*, 1631, published in Latin (English edition 1647), *Circles of Proportion*, 1632, and *Trigonometrie*, 1657. Oughtred laid extraordinary emphasis upon the use of mathematical symbols, altogether he used over 150 of them. Three have survived to the modern times, namely the *cross* symbol for multiplication, $::$ as that of proportion, and the symbol for "*difference between*".

Leibniz (1646-1715) had serious, logical doubts and reservations to the use of Oughtred's cross symbol because of possible confusion with the letter X. On 29 July 1698 he wrote in a letter to John Bernoulli : "*I do not like (the cross) as a symbol for multiplication, as it is easily confounded with x; often I simply relate two quantities by an*

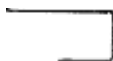
interposed dot and indicate multiplication by ZC.LM."

Symbol for inequality

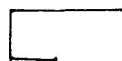
Thomas Harriot (1560-1621) was an English mathematician who lived the longer part of his life in the sixteenth century but whose outstanding publication appeared in the seventeenth century. His great work in this field, the *Artis Analyticae Praxis* was published in London posthumously in 1631, and deals largely with the theory of equations. In it he makes use of these symbols, ">" for "is greater than", and "<" for "is less than"

While Harriot was surveying North America, he saw a Native American with this symbol on his arm ~~X~~. it is likely he developed the two symbols from this symbol.

They were not immediately accepted, for many writers preferred the symbols (shown below), which another Englishman William Oughtred (1574-1660) had suggested in the same year in the popular *Clavis Mathematicae*, a work on arithmetic and algebra that did much toward spreading mathematical knowledge in that country.



Greater than



Less than

Symbol for percentage %

Percent has been used since the end of the fifteenth century in business problems such as computing interest, profit and

loss, and taxes. However, the idea had its origin much earlier. When the Roman emperor Augustus levied a tax on all goods sold at auction, *centesima rerum venalium*, the rate was 1/100. Other Roman taxes were 1/20 on every freed slave and 1/25 on every slave sold. Without recognising percentages as such, they used fractions easily reduced to hundredths

$$\frac{\text{O}}{\text{O}}$$

The percent sign, %, has probably evolved from the symbol shown above introduced in an anonymous Italian manuscript of 1425. Instead of "per 100," "P cento," which were common at that time, this author used the symbol shown.

TERMS

ADDITION Fibonacci used the Latin *additio*, although he also used *compositio* and *collectio* for this operation. The arithmetic word *add* is from the Latin root *addere*, to give or to do. The *dere* part of the root is the same root that gives us *Data/Datum* and the name for dice. *Donation* and *condone* also share the same root. The first recorded use of the word in English is from "*The Crafte of Nombrynge*". The document was one of the first English language documents dealing with mathematics.

COMPUTE It is joining of the *com* (with) and the Latin root *putare*. This root is often cited as related to thinking or reckoning, but its meaning comes from an early word for cut or slice. The same root appears in *amputate*. This goes

back to the earliest use of numbers in commerce and the idea of comparing values to a counting or tally stick. The sticks were notched to record values for future reference. Computing, then, was comparing the quantity to the marks on the tally stick

DIGIT The word digit refers both to the fingers (and toes) as well as the Arabic number symbols for 0 to 9. The root is the Indo-European word, *deik* and is related to many other words that reflect the use of the hands and fingers to "point" out objects. Index, indicate, dictate, indict, token, dice, judge and teach are all related to the same root.

CALCULATE / CALCULUS The origin of both these words is in the Greek word *kalyx*, for pebble or small stone. The manipulations of small stones on counting boards to do arithmetic operations led to the present mathematical meanings of calculate and calculus. The pebble root is still present in the medical use of the word calculus, a name for an accretion of mineral salts in the body into "stone" such as kidney stones.

DIFFERENCE It is the combination of two roots *dis* (away) and the second root, *ferre* is from the Latin for to carry. The difference between two numbers is the amount that one has been "carried away" from the other. The same root is present in fertile, but not in ferry.

DIVIDE shares its major root with the word **widow**. The root *vidua* refers to a separation. Widow is one who is separated from the spouse. The prefix, *di*, of divide is a contraction of *dis*, a two based word meaning apart or away, as in the process of division in which equal parts are

separated from each other. The *vi* part of *vidua* is also derived from a two word, and is the same root as in vigesimal (two tens), for things related to twenty. An individual is one who cannot be divided.

ARITHMETIC was the Greek word for number, and is closely related to the root of **reckon**, which is an obsolete term for count. In the middle ages the best mathematicians of Germany were called *Reichenmeister* and their arithmetic texts were *Reichenbucher*. The beginning of the word is drawn from the Indo-European root *ar* that is related to "fitting together" and gives us words like army, and art. Order, adorn, and rate all come from variants of the same root.

AVERAGE The meaning of **average**, as it is used in math today, comes from a commercial practice of the shipping age. The root, *aver*, means to declare, and the shippers of goods would declare the value of their goods. When the goods were sold, a deduction was made from each persons share, based on their declared value, for a portion of the loss, their **AVERAGE**.

FRACTION comes from the Latin word *frangere*, to break. A fraction represented the broken portion of whole.

HUNDRED is from the German root *hundert*. The quantity that it represents has not been consistent over the years and has ranged from its present value, 100, to 112, 120, 124, and 132 at different times in different areas. The remnants of these old measures still persist in the *hundredweight* of some countries representing 112 or 120 pounds, depending on the country.

MULTIPLY comes from the combined roots of *multi*, many, and *pli*, for folds, as in a number folded on itself many times.

THOUSAND Our number for one thousand comes from an extension of hundred. The roots are from the Germanic roots *teue* and *hundt*. *Teue* refers to a thickening or swelling, and *hundt* is the root of our present day hundred. A thousand, then, literally means a swollen or large hundred. The root *teue* is the basis of such common words today as thigh, thumb, tumor, and tuber.

Chapter 3 ETYMOLOGY OF ALGEBRA

The word *algebra* is a Latin variant of the Arabic word *al-jabr*. This came from the title of a book, *Hidab al-jabr wal-muqubala*, written in Baghdad about 825 A.D. by the Arab mathematician *Mohammed ibn-Musa al-Khowarizmi*.

The words *jabr* (JAH-ber) and *muqubalah* (moo-KAH-ba-lah) were used by al-Khowarizmi to designate two basic operations in solving equations. *Jabr* was to transpose subtracted terms to the other side of the equation. *Muqubalah* was to cancel like terms on opposite sides of the equation. In fact, the title has been translated to mean "science of restoration (or reunion) and opposition" or "science of transposition and cancellation" and "The Book of Completion and Cancellation" or "The Book of Restoration and Balancing."

Jabr is used in the step where $x - 4 = 16$ becomes $x = 20$. The left-side of the first equation, where x is lessened by 4, is "restored" or "completed" back to x in the second equation.

Muqabalah takes us from $x + y = y + 7$ to $x = 7$ by "cancelling" or "balancing" the two sides of the equation.

Eventually the *muqabalah* was left behind, and this type of math became known as algebra in many languages.

It is interesting to note that the word *al-jabr* used non-mathematically made its way into Europe through the

Moors of Spain. There an *algebrista* is a bonesetter, or "restorer" of bones. A barber of medieval times called himself an algebrista since barbers often did bone-setting and bloodletting on the side. Hence the red and white striped barber poles of today. The first use of the word "algebra" in English was by the Welsh mathematician and textbook writer, Robert Recorde in his Pathway of Knowledge written about 1550 .

Algebra is the heart of problem solving almost all problems except for the strictly geometric or logical, uses equations.

Chapter 4 WHY IS ELEVEN NOT ONETEEN AND TWELVE NOT TWOTEEN?

There would not be many mothers who did not have to face the music while making their wards memorize the count to twenty. The curious child often wondered on eleven and twelve. The most obvious thing was to call them *oneteen* and *twoteen*. Then why this strange notation.

Ten is the number of fingers on both hands so it was natural to develop it as the base for counting, since the most basic counting and calculation is done using the finger and thumb. But the decimal system has a small drawback that there are not many proper fractions of 10, just 2 and 5. It is felt that the duodecimal system of 12 had this advantage of having 2,3,4 and 6 as fractions.

Moreover the choice of the number twelve may have had the following reasons

- the approximate number of lunar months in an Earth year;
- the sum of ten fingers on human hands and two feet; or
- the number of phalanx bones in the four fingers of one hand, with the thumb used as an indicator.

The last reason seems to be the most authentic reason. Thus one after ten and two after ten developed as eleven and twelve. Eleven in Old English is *endleofan*, and related forms in the various Germanic languages point back to an original Germanic *ainlif*, “eleven.” *Ainlif* is composed of

ain-, “one,” the same as our one, and the suffix *-lif* from the Germanic root *lib-*, “*remain left over*.” Thus, eleven is literally “one-left” (over, that is, past ten), and twelve is “two-left” (over past ten).

In many civilisations, 12 the duodecimal system was adopted probably because there are 12 signs of the zodiac. There are 12 hours in a day or night. All traditional Chinese calendars, clocks and compasses have their basis in 12.

Many European languages have special words for 11 and 12. Being a versatile denominator in fractions may explain why we have 12 inches in a foot, 12 ounces in a troy pound, 12 old British pence in a shilling, 12 items in a dozen.

That the number twelve was important in ancient times is given evidence by the fact that, in the Germanic languages at any rate, number elements are not repeated until after the number twelve. For, of course, the literal meanings of the words eleven and twelve are “one left” and “two left” from the Germanic compounds “*ain-lif*” and “*twa-lif*.” So its just a tradition based on some requirement that 11 and 12 are what they are and not what we thought they should be.

Chapter 5 THE WORLD'S LARGEST NUMBER

This is certainly the most undeserving quest by any serious mathematician. When everybody has spoken so much about infinity and all related stuff, how we can possibly define the world's largest number.

But still it is worthwhile to go through some very very large numbers and what they mean.

Googol A number invented by the nine year old nephew of Dr Edward Kasner when asked to think of a name for a 1 followed by 100 zeros. 10^{100} is an incredibly large number. The largest reasonable estimate for the number of particles in the universe is only about 10^{85} . A googol is a million times a billion times this much.

$$\text{GOOGOL} = 10^{100}$$

The world's largest number, according to the Guinness Book of Records, is **Graham's number**. The number is named for Ronald L Graham, juggler, acrobat, and mathematician. Of course every good math student knows there can be no "largest" number. Actually the record book lists it as the largest number ever used in a proof. The number is incomprehensibly large, beyond images like the number of grains of sand needed to fill the universe or any such comparison. It cannot even be described with conventional numerical techniques. David Wells wrote in *The Penguin Dictionary of Curious and Interesting Numbers*, "If all the material in the universe was turned into pen and ink it

would not be enough to write the number down." To write the number a special notation invented by Donald Knuth is required, and even then it grows out of reason ...

3^3 means '3 cubed', as it often does in computer printouts. $3^{^3}$ means $3^{(3^3)}$, or 3^{27} , which is already quite large: $3^{27} = 7,625,597,484,987 \dots 3^{^{^3}} = 3^{(3^{^3})}$, however, is $3^{7,625,597,484,987}$ $= 3^{(7,625,597,484,987^{7,625,597,484,987})}$, which makes a tower of exponents 7,625,597,484,987 layers high. $3^{^{^{^3}}} = 3^{^{(3^{^{^3}})}}$, of course.

Graham's number only starts here. Consider the number $3^{^{^{^{\dots^{^3}}}}}$ in which there are $3^{^{^{^3}}}$ arrows. A largish number! Next construct the number $3^{^{^{^{\dots^{^3}}}}}$ where the number of arrows is the previous $3^{^{^{^{\dots^{^3}}}}}$ number !

A *googolplex*, is a number which is impossible to write down with ordinary numeration, since this would entail the digit "1" followed by a *googol* of zeroes.

$$10^{10^{100}}$$

GOOGOLPLEX

Chapter 6 *SIFR*-THE FATHER OF ZERO

It is widely believed that invention of Zero was one of most important events in Mathematics. The word Zero is strange and its evolution is even more interesting. The word *zero* comes from the Arabic *sifr* (رِفص) meaning *empty* or *vacant*, a literal translation of the Sanskrit *śūnya* meaning *void* or *empty*. Through transliteration this became *zephyr* or *zephyrus* in Latin.

In Latin *zephyrus* meant "west wind" ; the proper noun *Zephyrus* was the Roman god of the west wind (after the Greek god Zephyros). Since Zero came to be associated with it, *zephyr* came to mean a light breeze—"an *almost* nothing". The word *zephyr* survives with this meaning in English today. The Italian mathematician Fibonacci (c.1170-1250), who grew up in Arab North Africa and is credited with introducing the Arabic decimal system to Europe, used the term *zephyrum*. This became *zefiro* in Italian, which was slendered to *zero* in the Venetian dialect, giving the modern English word.

Words derived from *sifr* and *zephyrus* came to refer to calculation gradually as the decimal zero and its new mathematics penetrated through Europe. In thirteenth-century Paris, a 'worthless fellow' was called a... *cifre en algorithme*, i.e., an 'arithmetical nothing.' " (*Algorithm* is also a borrowing from the Arab, in this case from the name of the 9th-century Arab mathematician *al-Khwarizmi*.) This Arabic root gave rise to the modern French *chiffre*, which means digit, figure, or number; *chiffre*, to calculate or compute; and *chiffre*, encrypted; as well as to the English

word *cipher*. Today, the word in Arabic is still *sifr*, and similars of *sifr* are common throughout the languages of Europe. A few additional examples follow.

- **Polish:** *cyfra*, digit; *szyfrować*, to encrypt
- **German:** *Ziffer*, digit, figure, numeral, cypher
- **French:** *zéro*, zero
- **Spanish:** *cifra*, figure, numeral, cypher, code; *ceró*, zero
- **Swedish:** *siffra*, numeral, sum, digit

How can Greek be forgotten? Zero in Greek is translated as *Μηδέν* (**Meithen**).

Zero as a decimal digit

The earliest known *decimal digit* zero was probably introduced by Indian mathematicians sometime around the 3rd century. It was written in the shape of a dot, and consequently called *bindu* "dot". An early documented use of the zero by *Brahmagupta* dates to 628. He treated zero as a number and discussed operations involving it. By this time (7th century) the concept had clearly reached Cambodia, and historians believe that these ideas later spread to China and the Islamic world.

The Hindu-Arabic number system reached Europe in the late 11th century. The Italian mathematician Fibonacci was instrumental in bringing the system into European mathematics around 1200, though he spoke of the "sign" zero, not as a number.

***Chapter 7* WHY THERE IS NO NOBEL PRIZE IN MATHEMATICS?**

Those of us who have followed the annual ceremony of announcement of awards for the world's most promising scientists, technologists, economists, peace brokers, the NOBEL prize, may have wondered why after all mathematics has been left out in the fray.

Some weird story goes that Swedish mathematician Gosta Magnus Mittag-Leffler had run off with Alfred Nobel's wife. Supposedly, later in revenge Nobel refused to endow one of his prizes in mathematics. But this story is beyond credence, as Alfred Nobel never really married. One more anecdote associated is that Mittag-Leffler, antagonized Nobel by his fame and wealth. Nobel perhaps did not want Mittag-Leffler as the leading Swedish mathematician to win a Nobel Prize in mathematics, so he declined to institute a prize in Mathematics.

Some scholars are of the opinion that Mittag-Leffler and Nobel had almost no relation to each other; Nobel emigrated from Sweden in 1865 when Mittag-Leffler was a student and rarely returned to visit. Perhaps the idea for Nobel prize did not click with Alfred Nobel quite naturally as Mathematics did not directly affect the lives of people and he was more of a technologist.

Nobel's final will of 1895 bequeathed \$9,000,000 for a foundation whose income would support five annual prizes in physics, chemistry, medicine-physiology, literature, and peace. Four of the original five prizes were in fields, which

were close to Nobel's own interests, medicine being the exception.

A sixth Nobel Prize in economic science was added in 1969. The addition of this new Nobel Prize suggests the possibility at some future date of a seventh Nobel Prize. A strong case is there for a new Nobel Prize in the mathematical sciences, which could include statistics, computer science, etc. The Nobel equivalent is the Fields Medals that are awarded at each International Congress of Mathematicians. But these are given only every four years to a mathematician under forty, and not as famous as the other awards.

Chapter 8 MYSTERIOUS INFINITY

*There was a young fellow from Trinity
Who took the square root of infinity
but the number of digits
gave him the fidgets;
He dropped Math and took up Divinity.*

The symbol for infinity

John Wallis (1616-1703) was one of the most original English mathematicians of his day. He was educated for the Church at Cambridge and entered Holy Orders, but his genius was employed chiefly in the study of mathematics. The *Arithmetica Infinitorum*, published in 1655, is his greatest work.



This symbol for infinity is first found in print in his 1655 publication *Arithmetica Infinitorum*. It may have been suggested by the fact that the Romans commonly used this symbol for a thousand, just as today the word “myriad” is used for any large number, although in the Greek it meant ten thousand. The symbol was used in expressions such as, in 1695, “jam numerus incrementorum est (infinity).”

The symbol for infinity, first chosen by John Wallis in 1655, stands for a concept that has given mathematicians

problems since the time of the ancient Greeks. A case in point is that of Zeno of Elea (in southern Italy) who, in the 5th century BC, addressed whether magnitudes (lengths or numbers) are infinitely divisible or made up of a large number of small indivisible parts.

A very boring way to start with the mysterious term is to call it “without limit”. The first thing that the non-curious mind asks, “If there is no limit, then lets study within limits. As it is whatever is in syllabus is INFINITELY tough and dull.”

There is no record of earlier civilizations regarding conceptualizing or discussing infinity, but the story of infinity can begin with the ancient Greeks. Originally the word *apeiron* meant unbounded, infinite, indefinite, or undefined. It was a negative, even pejorative word. For the Greeks, the original chaos out of which the world was formed was *apeiron*. Aristotle thought being infinite was a privation not perfection.

The concept of infinity was forced upon the Greeks from the physical world by three traditional observations.

- Time seems without end.
- Space and time can be unendingly subdivided.
- Space is without bound.

Aristotle says the infinite is imperfect, unfinished and unthinkable.

An interesting observation with infinite series is that they are not very well behaved.

Let us *feel* this series

$$1 - 1 + 1 - 1 + 1 - 1 + \dots$$

If the terms are grouped this way,

$$(1 - 1) + (1 - 1) + (1 - 1) + \dots,$$

then the sum appears to be

$$0 + 0 + 0 + \dots = 0.$$

But if the terms are grouped differently,

$$1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \dots,$$

then the sum appears to be

$$1 + 0 + 0 + 0 + \dots = 1.$$

So $0=1$. It would be incorrect to conclude that $0 = 1$.

Instead, we conclude that infinite series do not always obey the traditional rules of algebra, especially those that permit the arbitrary regrouping of terms.

Let us look at the series

$$1 + 1/2 + 1/4 + 1/8 + 1/16 + \dots$$

the sum always is less than 2 but keeps increasing as more terms are added, although it approaches nearer and nearer

to 2 as more terms are included. On the other hand, the series

$$1 + 1/2 + 1/3 + 1/4 + 1/5 + \dots$$

is called divergent: it has no limit, the sum becoming larger than any chosen value if sufficient terms are taken.

The paradox is the fact that there are just as many even natural numbers in the first series as there are even and odd numbers altogether in the second series, thus contradicting the notion that “*the whole is greater than any of its parts.*” This seeming contradiction arises from the properties of collections containing an infinite number of objects. Since both are infinite, they are for both practical and mathematical purposes equal.

Consider the following **sequence** or list of numbers:

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n}, \dots$$

This is the sequence whose n^{th} **term** is $1/n$. Now we say that the sequence has the limit 0 as n goes to infinity,

$$\frac{1}{n} \rightarrow 0 \text{ as } n \rightarrow \infty$$

It connotes that as we go further into the series all terms get closer to zero.

From this we get the notion of an **infinite series**.

Consider the **partial sums**

1

$$1 \frac{1}{2} = 1 + \frac{1}{2}$$

$$1 \frac{3}{4} = 1 + \frac{1}{2} + \frac{1}{4}$$

$$1 \frac{7}{8} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

...

$$1 \frac{2047}{2048} = 1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2048}$$

and so on. Now these sums are headed somewhere, namely, toward 2. If you add on enough terms, eventually you will get as close as you like to 2. So we say that the sum of all the terms is 2, that is

$$\sum_{n=0}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 2$$

We say that an infinite series with a sum **converges**. Some series don't converge, like $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$, the **harmonic series**. To see that the partial sums go off to infinity, note that we have 1 and $\frac{1}{2}$ and then

$$\frac{1}{3} + \frac{1}{4} > \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} > \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2},$$

the next 8 terms,

$$\frac{1}{9} + \frac{1}{10} + \dots + \frac{1}{16} > 8 * \frac{1}{16} = \frac{1}{2}$$

and so on. So our sum grows larger than any multiple of $1/2$. We say that the harmonic series **diverges**. A question may arise in the reader's mind, can infinity be halved. It must have an end. Is it the limitation of the human mind or a question GOD never wanted to answer nor let us know about?

Infinity also has a set of rules for arithmetical operations

With itself

$$\infty + \infty = \infty$$

$$\infty \times \infty = \infty$$

$$-\infty + (-\infty) = -\infty$$

$$-\infty \times (-\infty) = \infty$$

$$\infty \times (-\infty) = -\infty$$

There are certain undefined operations

$$0 \times \infty$$

$$\infty + (-\infty)$$

$$\frac{\infty}{\infty}$$

$$\frac{\infty}{\infty}$$

$$\frac{\infty}{\infty}$$

It is important to know that $x/\infty = \mathbf{0}$ is not equivalent to $\mathbf{0} \cdot \infty = x$. If this were true, then it would have to be true for **every** x , which would mean all numbers are equal, an impossible proposition. Thus $\mathbf{0} \cdot \infty$ remains undefined, or *indeterminate*.

Last word- *Hotel Infinity*

The great mathematician David Hilbert, often asked to explain the curious nature of infinity, once developed a novel thought experiment to shed light upon the mystery:

Imagine a hypothetical hotel with an infinite number of rooms. One day a new guest arrives and is disappointed to learn that, despite the hotel's infinite size, it has no vacancies. Fortunately the clerk (Hilbert) has a solution. He simply asks each of the guests to move to the next room: the guest in room 1 moves to room 2, the guest in room 2 to room 3, and so on. This allows the new arrival to slip into the newly vacant room (1). So far so good...

The following night, however, Hilbert is presented with a more challenging problem - the arrival of an infinitely large number of new guests. Hilbert, delighted by the prospect of infinitely more hotel bills, once again has a solution. He simply asks each guest to move to the room whose number is twice that of his or her current room: the guest in room 1 moves to room 2, the guest in room 2 moves to room 4, and so on. Everyone still has a room but an infinite number of rooms (all the odd ones and then some) have been vacated for the new arrivals!

Chapter 9 MISUNDERSTOOD CALCULUS

The word “calculus” stems from the nascent development of mathematics: the early Greeks used pebbles arranged in patterns to learn arithmetic and geometry, and the Latin word for “pebble” is “calculus,” a diminutive of *calx* (genitive *calcis*) meaning “limestone”.

The word Calculus sends shivers in the back bones of students, blanks off their minds with huge *greeky* equations. It is probably the most important development in mathematics applied in Physics, for the study of all natural systems and mathematical modelling and as a general method whenever the goal is an optimum solution to a problem that can be given in mathematical form

Calculus is built on two major complementary ideas. The first is **differential calculus**, which studies the rate of change in one quantity relative to the rate of change in another quantity. This can be illustrated by the slope of a line. The second is **integral calculus**, which studies the accumulation of quantities, such as areas under a curve, linear distance traveled, or volume displaced.

The simplest definition that can be offered is that **Calculus is the study of mathematically defined change.**

Examples of typical differential calculus problems include:

- finding the acceleration and speed of a free-falling body at a particular moment

- finding the optimal number of units a company should produce to maximize their profit.

Examples of integral calculus problems include:

- finding the amount of water pumped by a pump with a set power input but varying conditions of pumping losses and pressure
- finding the amount of grass mowed by a lawn mower of given power with varying density of grass.

Newton was a physicist and he needed to know the speed of an object moving in one dimension. He invented calculus for this purpose. Speed is simply the distance traveled by time taken. But in advanced science using gravity as the force, the function is continuous and not discrete. Thus the velocity of the object is derivative of x , the distance with respect to time t .

$$v = \frac{dx}{dt} \text{ which is actually } \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t},$$

here Δx and Δt are infinitesimal distance and time respectively.

Suppose you were asked to find out the rate at which the universe is expanding with the following information in hand,

Let t be the time that has elapsed since the Big Bang. In that time, light, traveling at speed c , has been able to travel

a maximum distance ct . The portion of the universe that we can observe is therefore a sphere of radius ct , with volume,

$$v = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (ct)^3$$

The differential calculus will tell us to differentiate velocity, which is

$$\frac{dv}{dt} = \frac{4}{3} \pi c^3 .3t^2$$

Without going into details of rules for differentiation, we find that rate of expansion is proportional to $c^3.t^2$, huge, you can well imagine the enormous rate at which we are expanding! Next let us see what integral calculus has to do with our lives.

Work is a measure of the amount of energy transferred by a force. If the locomotive sets the wagon in motion, the loco's force on the wagon transfers energy to the wagon. If F is the force applied on the wagon and it moves by distance x , then to find out the work done we will have to multiply the force F , several times in very small intervals of time. These very small values are called infinitesimal.

Here we use the integration of this infinitesimal work done to find out the total work done.

If dW is infinitesimal work done and dx is the infinitesimal distance, then we have $dW = F . dx$

If the wagon moved from position a to b then total work done, W is

$$W = \int_a^b F \cdot dx$$

This is integral calculus.

Calculus, in fact is the easiest of all branches. It is a straightforward thing, can be visualized easily. Next time don't be scared.

The restricted meaning of *calculus*, meaning differential and integral calculus, is due to Leibniz. Newton did not originally use the term, preferring *method of fluxions*. He used the term *Calculus differentialis* in a memorandum written in 1691.

The story goes that both Newton and Leibniz independently developed calculus. But the credit for invention and propagation of calculus goes to Newton.

Chapter 10 FOUR FOURS

The reader must have got slightly bored with injections of “knowledge”, here is something not to worry about. I have been told that there is a way using four fours and any mathematical expression to write all numbers from 1 to 100.

Here are a few examples, rest all for you to discover.

1	$(4+4-4)/4$
2	$(4 \times 4)/(4+4)$
4	$(4-4) \times 4+4$
14	$4+4+4+\sqrt{4}$
25	$4!+\sqrt{4-4/4}$
50	$44+(4!/4)$
82	$4 \times (4!-4)+\sqrt{4}$
100	$4 \times 4!+\sqrt{4}+\sqrt{4}$

You can use all arithmetical operators. Special operators like factorial, square root and raise to power are all allowed. Interestingly there is more than one way to express the numbers.

Chapter 11 MULTIPLICATION TABLES

I proceeded for the most frightful experience of my life.

Multiplication table of Seven. Why seven you must be wondering, because it has apparently no pattern being mid way to 5 and 10. But as system would have had it, I crammed up tables for 1,2,3,4,5,6,7,8,9 and 10. I still wonder the need for this exercise, as there is hardly a calculation done in real life using these tables. With advent of calculators in watches, address diaries, cell phones this could be reviewed. The Pedagogue's justification for the most boring arithmetical task,

"The multiplication tables are a terrific tool for building the skill of memorization essential to learning during the grammar stage of the educational process. "

Some interesting patterns in tables

Look at the first digits of table of 9, they are increasing from 0 to 9.

The second digit is decreasing from 9 to 0. So no need to memorize this table just write the numbers

0	1	2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2	1	0

There you have the table of nine

By using the famous deductive technology of mathematical learning. Add to all digits in table of 5 in increasing order starting from top – 2,4,6,8,10,12,14,16,18,20 to get the table of 7.

Here is something for multiplying nine by seven. Count the seventh finger from the right and bend it:



There are six fingers to the right of the bent finger, and three fingers to the left. So we have $9 \times 7 = 63$. It works!

Some simple facts

- To multiply a number by five, multiply a half of that number by ten
- When you multiply a number by two, you just add the number to itself
- To multiply numbers that differ by two, multiply the number between them by itself and subtract one.

Interesting property of table of 8

	Integer sum		
$8 \times 1 = 8$	8	8	
$8 \times 2 = 16$	$1+6=7$	7	
$8 \times 3 = 24$	$2+4=6$	6	
$8 \times 4 = 32$	$3+2=5$	5	
$8 \times 5 = 40$	$4+0=4$	4	
$8 \times 6 = 48$	$4+8=12=1+2=3$	3	
$8 \times 7 = 56$	$5+6=11=1+1=2$	2	
$8 \times 8 = 64$	$6+4=10=1+0=1$	1	
$8 \times 9 = 72$	$7+2=9$	9	
$8 \times 10 = 80$	$8+0=8$	8	
$8 \times 11 = 88$	$8+8=16=1+6=7$	7	
$8 \times 12 = 96$	$9+6=15=1+5=6$	6	

Do you see the pattern, sequence 9 to 1 repeats.

I wish it was known to me before

Similarly for table of 6

	Integer sum		
$6 \times 1 = 6$	6	6	
$6 \times 2 = 12$	$1+2=3$	3	
$6 \times 3 = 18$	$1+8=9$	9	
$6 \times 4 = 24$	$2+4=6$	6	
$6 \times 5 = 30$	$3+0=3$	3	
$6 \times 6 = 36$	$3+6=9$	9	
$6 \times 7 = 42$	$4+2=6$	6	
$6 \times 8 = 48$	$4+8=12=1+2=3$	3	
$6 \times 9 = 54$	$5+4=9$	9	
$6 \times 10 = 60$	$6+0=6$	6	
$6 \times 11 = 66$	$6+6=12=1+2=3$	3	
$6 \times 12 = 72$	$7+2=9$	9	

Do you see the pattern, 6,3,9 repeats.

Its an easy way to remember.

Chapter 12 DIVISIBILITY FUNDA

A very important aspect of Arithmetic in School is the rule for divisibility. There are standard rules for numbers like 2,3,4,5,6,8,9,11 etc. But for numbers like 7,12 and so on no rules are found in textbooks. Some great mathematician has simplified things and there is a rule for almost all numbers, which we shall explore in this chapter.

Schooling teaches a few basic rules which are summarized thus,

2	If the last digit divisible by two, then number is too
3	If the sum of the digits of the number is divisible by three, then number is too
4	If the last two digits are divisible by four, then is too
5	If the last digit is 5 or 0, then is divisible by 5
6	If is divisible by 2 and by 3, then number is divisible by 6
8	If the last three digits are divisible by 8, then number is too
9	If the sum of the digits of the number is divisible by nine, then is too
10	If the last digit is 0, then number is divisible by 10

The missing rule is for seven. The teacher told us seven has no rule for divisibility. I kept wondering and pondering till it was found. A bit different but good.

This rule is called L-2M.

The last digit from the number is doubled and subtracted from the remaining number; the procedure is repeated till some number is obtained which is identified as being divisible by 7.

Suppose the number is 6125

$612 - (5 \times 2) = 612 - 10 = 602$ **Step**
here 5 last digit is doubled and subtracted, now use **1**
602 where 2 the last digit should be doubled and
subtracted from 60.

$60 - (2 \times 2) = 60 - 4 = 56$ which is 7×8 , **Step**
 hence 6125 is divisible by 7. **2**

Try a few numbers, and then you would really appreciate the technique.

The same rule is extendable to 13, 17 and 19.

For 13	$L+4M$
For 17	$L-5M$
For 19	$L+2M$

Suppose we were to test divisibility of 9994 with 19, using $L+2M$ which means last digit is doubled and *added* to remaining number.

$999 + 8 = 1007$ **Step 1**

$100 + 14 = 114$ **Step 2**

$11 + 8 = 19$ itself so 9994 is divisible by 19 **Step 3**

In Vedic mathematics there is a divisibility rule for almost any number. It is the method of osculation using the *Veṣṭana*, finding the *Ekādhika* of the divisor. We shall not go into the details for calculating the *Ekādhika* of the divisor, but explain the procedure for checking divisibility.

Let us understand what osculation means,

Suppose you were to osculate 21 with 5, then multiply the last digit with 5 i.e. $5 \times 1 = 5$, add this to previous digit 2 and thus get 7. This is the simplest of osculations.

A few more illustrative examples.

4321 with 7 –

$432 + 7 = 439$, $43 + 7 \times 9 = 106$, $10 + 7 \times 6 = 52$, $5 + 7 \times 2 = 19$, $1 + 7 \times 9 = 64$ and so on to get 34,31,10,1

The purpose of osculation is to reach a number, which is easily identifiable as divisible, or not.

Firstly the *Ekādhika* for the numbers.

1. For 9,19,29,39, ... they are 1,2,3,4,.....respectively
2. For 3,13,23,33, ... they are 1,4,7,10,.....respectively
3. For 7,17,27,37, ... they are 5,12,19,26...respectively
4. For 1,11,21,31, ... they are 1,10,19,28...respectively

OSCULATION BY THE *Ekādhika*

Let us again check the divisibility of 9994 by 19

- | | | |
|-------|--|--|
| (i) | The <i>Ekādhika</i> for 19 is 2, so multiply the last digit by 2 and add the product 8 to previous digit 9. | 9994
$4 \times 2 = 8$
$8 + 9 = 17$
99 left |
| (ii) | Now osculate 17 with 2 to get 15, add 15 with the next right most digit, 9 to get 24. | 17
$1 + 7 \times 2 = 15$
$15 + 9 = 24$
9 left |
| (iii) | Now osculate 24 with 2 to get 10, add 10 to the left most digit 9, to get 19 which is a multiple of 19, hence 9994 is divisible by 19. | 24
$2 + 4 \times 2 = 10$
$10 + 9 = 19$ |

This is one way the other way is to simply osculate with the *Ekādhika*. The method has to be chosen with ease of calculation; sometimes the above method is simpler

Do you realize how simple it is, the only thing to remember is the osculator or the *Ekādhika*. If properly observed even that is simple to remember with a few mnemonics. This is perhaps the greatest contribution of Vedic mathematics.

Let us find out whether 7755 is divisible by 33 using the straight osculation method.

The *Ekādhika* for 33 is 10. Thus for 7755

$$775 + 50 = 825 \Rightarrow 82 + 50 = 132 \Rightarrow 13 + 20 = 33$$

This is simply terrific, must for all middle school students. But there is a slight problem if you see for numbers ending with 1 and 7 the process may become cumbersome, as the *Ekādhika* is a higher order number like 5,12,19,26 etc. For this another brilliant solution is the negative osculation method. Here the principle is same except that there is subtraction instead of addition and if the end result is a multiple of the number or zero (*which is true for most cases*) the divisibility is proved.

If P is the positive osculator and N is the negative osculator, then the rule is divisor, $D = P+N$.

For number 7, $P=5$, thus $N=7-2=5$

For number 21, $P=19$, thus $N=21-19=2$

The Negative osculators for the numbers.

For 7,17,27,37, ... they are 2,5,8,11.....respectively

For 11,21,31,41, .. they are 1,2,3,4.....respectively

To remember multiply the number to get a product ending in 1, remove the 1 and the remaining number is the negative osculator.

The important thing to remember here is that unlike the positive osculation here osculate by **subtracting** product of ***Ekādhika and the last digit***.

Let us consider example of testing divisibility of 165763 with 41

The negative osculator is 4.

$$16576 - 4 \times 3 = 16564$$

$$1656 - 4 \times 4 = 1640$$

$$164 - 4 \times 0 = 164$$

$$16 - 4 \times 4 = 0$$

Let us consider another example of 10171203 by 67, here multiply 67 by 3 which is 201, hence negative osculator is 20.

$$1017120 - 20 \times 3 = 1017060$$

$$101706 - 20 \times 0 = 101706$$

$$10170 - 20 \times 6 = 10050$$

$$1005 - 20 \times 0 = 1005$$

$$100 - 20 \times 5 = 0$$

Thus it is divisible

Here you see how easy it has become with simple arithmetic. Happy dividing!

Chapter 13 PRINTER'S ERRORS

I found a very interesting book on mathematical recreations, H. E. Dudeney's *Amusements in Mathematics*. All his recreations impressed me but this particular one called PRINTER's error is remarkable.

A printer when required to set the type for number $2^5 \cdot 9^2$, mistakenly set it as 2592 (the dot was meant to indicate multiplication). However, upon proofreading the number, it was found to be correct as written.

Because $2^5 \cdot 9^2 = 2592$

During the course of reading, a few more errors were discovered.

$$\begin{aligned}
 1129\frac{1}{3} &= 11^2 \cdot 9\frac{1}{3} \\
 2124\frac{9}{11} &= 21^2 \cdot 4\frac{9}{11} \\
 34425 &= 3^4 \cdot 425 \\
 312325 &= 31^2 \cdot 325
 \end{aligned}$$

Wait a while, try finding some more like these, it is a meditative experience. Some interesting *Dottable fractions* which are a kind of **Printer's error** follow,

$$\frac{416}{21879} = \frac{4.16}{2.187.9}$$

$$\frac{666}{64676} = \frac{6.6.6}{6.46.76}$$

$$\frac{388}{485} = \frac{3.8.8}{48.5}$$

These are a few *dottable fractions* but there are several such fractions. It would be interesting to find where to place the dots in the following fractions to make them dottable.

12980 / 74635
 13680 / 29754
 13950 / 46872
 17460 / 39285
 18630 / 27945
 32160 / 97485
 34560 / 91728
 46350 / 12978
 54270 / 18693
 78360 / 21549
 86310 / 92475
 92460 / 37185

Using the beast number

666/1998
 666/4995
 666/3478
 666/27972
 666/38665

Chapter 14 DIGITAL VARIANTS

This chapter is dedicated to some amusing types of numbers that are really interesting to explore and observe. I don't even remember the sources I got them from, as I used to pen them down in my diary.

Observe the numbers below

$$\begin{aligned}
 12\ 33 &= 12^2 + 33^2 \\
 990\ 100 &= 990^2 + 100^2 \\
 9412\ 2353 &= 9412^2 + 2353^2 \\
 74160\ 43776 &= 74160^2 + 43776^2 \\
 116788\ 321168 &= 116788^2 + 321168^2
 \end{aligned}$$

The sum of the squares of the two halves of the number is equal to the number itself.

I later found that there exist numbers as difference of squares of their halves, e.g. $48 = 8^2 - 4^2$

Can you find some more numbers like this?

The number maniac's obsession does not end here.

$$\begin{aligned}
 22\ 18\ 59 &= 22^3 + 18^3 + 59^3 \\
 166\ 500\ 333 &= 166^3 + 500^3 + 333^3
 \end{aligned}$$

And this one is even more complex

$$336 = (3^1 + 3^1 + 6^1) + (3^2 + 3^2 + 6^2) + (3^3 + 3^3 + 6^3)$$

Factorial is mathematical expression such that

$$n! = 1 \times 2 \times 3 \times 4 \dots (n-1) \times n$$

Does it not seem impossible that a number can be sum of factorial of its digits, but look below

$$145 = 1! + 4! + 5!$$

$$40585 = 4! + 0! + 5! + 8! + 5!$$

Alas,

$$4! + 1 = 5^2 \qquad 5! + 1 = 11^2 \qquad 7! + 1 = 71^2$$

These are called BROWN's number, and there can be more of these which will be fascinating to find.

Look at these

$$4150 = 4^5 + 1^5 + 5^5 + 0^5$$

$$4151 = 4^5 + 1^5 + 5^5 + 1^5$$

$$194979 = 1^5 + 9^5 + 4^5 + 9^5 + 7^5 + 9^5$$

These are called *Perfect Digital Invariants*, a PDI is a number equal to the sum of a power of its digits when the *power is not equal to the length of the number*. An interesting pattern called *Pluperfect Digital Invariants* or PPDIs is found next. They are also called Armstrong

Numbers. *In each case, the power corresponds to the number of digits.*

$$153 = 1^3 + 5^3 + 3^3$$

$$1634 = 1^4 + 6^4 + 3^4 + 4^4$$

$$54748 = 5^5 + 4^5 + 7^5 + 4^5 + 8^5$$

$$548834 = 5^6 + 4^6 + 8^6 + 8^6 + 3^6 + 4^6$$

$$1741725 = 1^7 + 7^7 + 4^7 + 1^7 + 7^7 + 2^7 + 5^7$$

The following numbers are unique in the sense that they use the numbers contained to reproduce themselves through mathematical operators, a very intriguing experience to even contemplate on them. They are called *repdigit Friedman's number*.

$$11111111111 = ((11-1)^{11} - 1*1) / (11-1-1)$$

$$2222222222222 = (2((22-2)/2)^{2^{2+2}} - 2) / (2+2/2)^2$$

(here observe the symbol ^ which means to the power of)

$$333333333 = ((3*3 + 3/3)^{3*3} - 3/3) / 3$$

$$444444444444444 = (4(44/4 - 4/4)^{4*4-4/4} - 4) / (4 + 4 + 4/4)$$

$$5555555555 = (5(5+5)^{5+5} - 5) / (5 + 5 - 5/5)$$

$$666666666666666 = (6((66-6)/6)^{6 + (66-6)/6} - 6) / (6 + (6+6+6)/6)$$

$$77777777777777 = (7((77-7)/7)^{7+7} - 7 + 7 - 7) / (7 + (7+7)/7)$$

$$88888888888888 = (8((88-8)/8)^{8+8-(8+8)/8} - 8) / (8 + 8/8)$$

$$99999999 = (9 + 9/9)^{9-9/9} - 9/9$$

A **Friedman number** is a positive integer which can be written in some non-trivial way using its own digits, together with the symbols $+$ $-$ $\times$ $/$ $^$ $()$ and concatenation. Friedman's number are basically the same stuff some are perfect and some normal. The difference will be evident by the following examples,

$$64550 = (6^4 - 5) \times 50 \quad \text{Perfect}$$

$$16875 = 1 \times 68 + 7^5 \quad \text{Perfect}$$

$$25872 = 528 \times 7^2 \quad \text{Normal}$$

$$37875 = 75 \times (8^3 - 5) \quad \text{Normal}$$

The above two cases have the digits used in order of their formation, hence perfect. This is a very common number there are about 75 such numbers below 9999. A few of them are,

25, 1206, 2187, 2500, 3125, 4096, 4628, 9261

Can you find out how? Believe me it's an interesting exercise.

Try the Friedman's number using all nine digits,

$$123456789 = ((86 + 2 \times 7)^5 - 91) \div 3^4$$

$$987654321 = (8 \times (97 + 6/2)^5 + 1) \div 3^4$$

Some more interesting numbers are

$$8833 = 88^2 + 33^2$$

$$9474 = 9^4 + 4^4 + 7^4 + 4^4$$

$$594 = 1^5 + 2^9 + 3^4$$

$$732 = 1^7 + 2^6 + 3^5 + 4^4 + 5^3 + 6^2 + 7^1$$

(Observe the powers and base)

$$1033 = 8^1 + 8^0 + 8^3 + 8^3$$

$$3413 = 1^1 + 2^2 + 3^3 + 4^4 + 5^5$$

$$3435 = 3^3 + 4^4 + 3^3 + 5^5$$

The following set of numbers is called the Common Base number.

Notice the base is common and powers coincide with the digits of the number.

$$4624 = 4^4 + 4^6 + 4^2 + 4^4$$

$$1033 = 8^1 + 8^0 + 8^3 + 8^3$$

$$595968 = 4^5 + 4^9 + 4^5 + 4^9 + 4^6 + 4^8$$

$$3909511 = 5^3 + 5^9 + 5^0 + 5^9 + 5^5 + 5^1 + 5^1$$

$$13177388 = 7^1 + 7^3 + 7^1 + 7^7 + 7^7 + 7^3 + 7^8 + 7^8$$

$$52135640 = 19^5 + 19^2 + 19^1 + 19^3 + 19^5 + 19^6 + 19^4 + 19^0$$

The beauty of numbers lies here. Some seemingly obscure number can be represented in amazing ways. Above are a few ways of looking at numbers. Searching for Friedman's number is a truly intriguing experience as each number can be calculated in several different ways. These numbers can also be found for base systems other than decimal system.

Dean, to the physics department. "Why do I always have to give you guys so much money, for laboratories and expensive equipment and stuff.

Why couldn't you be like the mathematics department - all they need is money for pencils, paper and waste-paper baskets.

Or even better, like the philosophy department. All they need are pencils and paper."

Chapter 15 0123456789 AND MORE

So much about numbers, but what fascinates me the most is the use of all numbers 0 to 9. There are so many things, which can be done using ALL the numbers.

I could find two ways of getting 100 by placing any mathematical operator i.e $+$ $-$ $\times$ $/$ $()$.

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \times 9 = 100$$

$$1 + 2 \times 3 + 4 \times 5 + 6 + 7 \times 8 + 9 = 100$$

There are atleast four more ways to do so, lets see if you can do it.

Can you find the numbers which can do so ?

$$\begin{array}{r}
 123456789 \\
 \times \square\square\square\square\square\square\square\square \\
 \hline
 \square\square\square\square\square\square\square\square \quad 987654321
 \end{array}$$

Here is one very interesting problem .

Use the digits 0 to 9 in such a manner, that the resulting number has the property of being divisible in a manner that the starting from left, first digit is divisible by 1, the next two together by 2, the next three by 3 and so on till 10. That is if the number is $abcdefghij$, then the whole number is

divisible by 10, after removing the last digit i.e. j , nine digits are left and the resulting number is divisible by nine, so forth.

Hint: The digit at the 5th place will be 5 and 10th place digit will be 0.

Next one is elementary, my dear reader

Fill in the boxes with 1, 2, 3, 4, 5, 6, 7, 8, and 9 to make the multiplication equation work

$$\begin{array}{r} \square\square\square\square \\ \times \quad \square \\ \hline \square\square\square\square \end{array}$$

Here is another example of a nice discovery,

$$\begin{aligned} 85555^2 - 1 &= 7319658024 \\ 97777^2 - 1 &= 9560341728 \end{aligned}$$

In both theses cases the number when squared is ONE MORE than a number, which contains all ten digits.

There are certain numbers which are of the form $abc \times de = fg \times hi$ where 1 to 9 all digits are used, here is one

$$158 \times 23 = 3634 = 79 \times 46$$

There are at least two more known to me, find out more.

Another 1 to 9 puzzle!!

Using the ciphers 1 up to 9, three numbers (of three ciphers each) can be formed, such that the second number is twice the first number, and the third number is three times the first number.

Hint: There are four solutions, one of them is

192, 384, and 576

SUM AND PRODUCT

$876+429 = 1305$ is one way to write a sum which uses all digits (0-9) only once. How many different ways are there to do this? Wait, try and then read ahead.

$$879 + 426 = 1305$$

$$859 + 347 = 1206$$

$$789 + 264 = 1053$$

$$657 + 432 = 1089$$

$$756 + 342 = 1098$$

$$589 + 473 = 1062$$

The same question may be asked for products.

There are 22 solutions known, a few of them are elucidated.

$$58401 = 63 \times 927$$

$$19084 = 52 \times 367$$

$$16038 = 27 \times 594 \text{ or } 54 \times 297$$

$$65821 = 7 \times 9403$$

$$36508 = 4 \times 9127$$

$$27504 = 3 \times 9168$$

$$20754 = 3 \times 6918$$

Here are some interesting patterns

$$1 \times 8 + 1 = 9$$

$$12 \times 8 + 2 = 98$$

$$123 \times 8 + 3 = 987$$

$$1234 \times 8 + 4 = 9876$$

$$12345 \times 8 + 5 = 98765$$

$$123456 \times 8 + 6 = 987654$$

$$1234567 \times 8 + 7 = 9876543$$

$$12345678 \times 8 + 8 = 98765432$$

$$123456789 \times 8 + 9 = 987654321$$

$$0 \times 9 + 1 = 1$$

$$1 \times 9 + 2 = 11$$

$$12 \times 9 + 3 = 111$$

$$123 \times 9 + 4 = 1111$$

$$1234 \times 9 + 5 = 11111$$

$$12345 \times 9 + 6 = 111111$$

$$123456 \times 9 + 7 = 1111111$$

$$1234567 \times 9 + 8 = 11111111$$

$$12345678 \times 9 + 9 = 111111111$$

$$123456789 \times 9 + 10 = 1111111111$$

SPECIAL SQUARE NUMBERS

Smallest square number using all ten digits

$$1\ 026\ 753\ 849 = 32\ 043^2$$

$$9\ 814\ 072\ 356 = 99\ 066^2$$

A variant to an earlier problem

By only inserting '+' and '-' signs between 1 2 3 4 5 6 7 8 9 (in order), can you get all numbers between 1 to 100.

A few examples

$$1+2+3+4-5+6+7-8-9=1 \text{ and } 123-4-5-6-7+8-9=100$$

ANOTHER 1 to 9

There are atleast 30 different square numbers, which use each of the digits 1 to 9 just once. Try finding out some, believe me its interesting, you'll get to know several properties of numbers

Two examples are

$$\begin{aligned} 215\ 384\ 976 &= 14\ 676^2 \\ 743\ 816\ 529 &= 27\ 273^2 \end{aligned}$$

Look at this set of numbers

1	9	25	36	784
9	25	361	784	

Can you find something special?

Yes, they are squares individually, containing digits 1 to 9 just once.

Find some more such sequences.

Which two numbers, containing together all the nine digits, will, when multiplied together, produce another number (the *highest possible*) containing also all the nine digits? Zero not allowed.

$$8,745,231 \times 96 = 839,542,176$$

If I multiply 51,249,876 by 3 (thus using all the nine digits once, and once only), I get 153,749,628 (which again

contains all the nine digits once). Similarly, if I multiply 16,583,742 by 9 the result is 149,253,678, where in each case all the nine digits are used.

If we multiply 32547891 by 6, we get the product, 195287346.

100 mixed up with 123456789

Can you write 100 in the form of a mixed number, using all the nine digits once, and only once?

Here is one of them, $91 \frac{5742}{638}$

Actually there are 11 ways to do it including the example. Try before proceeding further.

$$96 \frac{2148}{537}$$

$$96 \frac{1752}{438}$$

$$96 \frac{1428}{357}$$

$$94 \frac{1578}{263}$$

$$91 \frac{7524}{836}$$

$$91 \frac{5823}{647}$$

$$82 \frac{3546}{197}$$

$$81 \frac{7524}{396}$$

$$81 \frac{5643}{297}$$

$$3 \frac{69258}{714}$$

Discussion and Solutions

$$\begin{array}{r}
 123456789 \\
 \times 989010989 \\
 \hline
 122100120 \ 987654321
 \end{array}$$

This number **3816547290** has the required unique property of being divisible by 10,9,8,7,6,5,4,3,2,1 by striking off the right most digits successively i.e. remove 0, it is divisible by 9, remove 7290 it is divisible by 6 (that is the number of digits remaining in the number) and so on.

3816547290	divisible by	10
381654729	divisible by	9
38165472	divisible by	8
3816547	divisible by	7
381654	divisible by	6
38165	divisible by	5
3816	divisible by	4
381	divisible by	3
38	divisible by	2
3	divisible by	1

There are two known solutions using all nine digits for

$$\begin{array}{r}
 \square\square\square\square \\
 \times \square\square\square\square \\
 \hline
 \square\square\square\square
 \end{array}
 \quad
 \begin{array}{l}
 1738 \times 4 = 6952 \\
 1963 \times 4 = 7852
 \end{array}$$

Three more 1 to 9 puzzles, each number is twice the previous one and the series uses only 1 to 9 once.

$$\begin{array}{l}
 219, 438, 657 \\
 273, 546, 819 \\
 327, 654, 981
 \end{array}$$

Chapter 16 MYSTIFYING 1729 AND RAMANUJAN

Srinivasa Ramanujan was one of India's greatest mathematical geniuses. He made substantial contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions, and infinite series.

Ramanujan was born in his grandmother's house in Erode, a small village about 400 km southwest of Madras (*now Chennai, India*). When Ramanujan was a year old his mother took him to the town of Kumbakonam, about 160 km nearer Madras. His father worked in Kumbakonam as a clerk in a cloth merchant's shop. In December 1889 he contracted smallpox.

At the age of five Ramanujan started his primary schooling at Kumbakonam although he attended several different primary schools before entering the Town High School in Kumbakonam in January 1898. At the Town High School, Ramanujan was to do well in all his school subjects and proved himself to be all round brilliant scholar. In 1900 he began to work on geometric and arithmetic series.

He also made significant contributions to the development of partition functions and summation formulas involving constants such as π . A child prodigy, he was largely self-taught in mathematics and had compiled over 3,000 theorems by the year 1914 when he moved to Cambridge. Often, his formulae were stated without proof and were only later proven to be true. His results have inspired a large amount of research and mathematical papers.

On 18 February 1918 Ramanujan was elected a fellow of the Cambridge Philosophical Society and then three days later, his name appeared on the list for election as a fellow of the Royal Society of London. This was the greatest honour that he would receive.

Ramanujan's home state of Tamil Nadu celebrates 22nd December (Ramanujan's birthday) as 'State IT Day', memorializing both the man, and his achievements, as a native of Tamil Nadu.

This is what GH Hardy, the famous mathematician observed, and it is the most famous anecdote related to Ramanujan and we all remember him for 1729.

I remember once going to see [Ramanujan] when he was lying ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. 'No,' he replied, 'it is a very interesting number; *it is the smallest number expressible as the sum of two cubes in two different ways.*'

Distinct property of 1729

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

1729 is the smallest number that can be expressed as the sum of two cubes in 2 distinct ways. Such numbers have been dubbed *taxicab numbers*.

1729 is the third Carmichael number, and a Zeisel number. It is a centered cube number, as well as a dodecagonal number, a 24-gonal and 84-gonal number.

Just facts, guide for the pure mathematician.

Just one last thing about 1729

$$\begin{aligned} 1 + 7 + 2 + 9 &= 19 \\ 19 \cdot 91 &= 1729 \end{aligned}$$

Add up the digits, reverse the sum, multiply both and get 1729 again !!

Something brilliant about Ramanujan's constant is that it is almost an integer.

$$e^{\pi\sqrt{163}} = 262,537,412,640,768,743.99999999999925....$$

Surprised! The mathematical constants e and π are *transcendental numbers*, that is, they can never be the roots of finite equations with rational coefficients. Yet, here we have a combination of e and π that is almost an integer.

Chapter 17 LILAVATI

Bhaskaracharya otherwise known as *Bhaskara* is probably the most well known mathematician of ancient Indian today. Bhaskara was born in 1114 A.D. according to a statement he recorded in one of his own works. He was from Bijjada Bida near the Sahyadri mountains. Bijjada Bida is thought to be present day Bijapur in Mysore state, now state of Karnataka, India. Bhaskara wrote his famous ***Siddhanta Siroman*** in the year 1150 A.D. It is divided into four parts; *Lilavati* (arithmetic), *Bijaganita* (algebra), *Goladhyaya* (celestial globe), and *Grahaganita* (mathematics of the planets).

Here is his famous riddle from *Lilavati*

O girl! out of a group of swans, $7/2$ times the square root of the number are playing on the shore of a tank. The two remaining ones are playing with amorous fight, in the water. What is the total number of swans?

Another one

***The woman's necklace broke.
A row of pearls mislaid.
One sixth fell to the floor.
One fifth upon the bed.
The young woman saved one third of them.
One tenth were caught by her lover.
If six pearls remained upon the string
How many pearls were there altogether?***

Solutions

The group of swans is 16.

Let x be the number of swans, thus

$$\frac{7}{2}\sqrt{x} + 2 = x, \text{ or } x - 2 = \frac{7}{2}\sqrt{x}$$

squaring both sides we have, $x^2 - 4x + 4 = \frac{49}{4}x$

$$x^2 + 4 - \frac{65}{4}x = 0, \text{ solving this we have, } x = 16 \text{ or } \frac{1}{4}$$

Thus number of swans can only be 16

The answer to the Women's necklace is 30 pearls.

Let x represent the original number of pearls on the necklace

$$6 + (x/6) + (x/5) + (x/3) + (x/10) = x$$

$$(6/x) + 1/6 + 1/5 + 1/3 + 1/10 = 1$$

$$(36/x) + 1 + 6/5 + 2 + 3/5 = 6$$

$$(180/x) + 15 + 9 = 30$$

$$(180/x) = 6$$

$$x = 30$$

Chapter 18 AMICABLE NUMBERS

Pythagoras, the famous Greek when asked, "What is a friend", replied that a friend is one "who is the other I" such as 220 and 284. The numbers 220 and 284 form the smallest pair of amicable numbers (also known as friendly numbers) known to Pythagoras.

Two numbers are called Amicable (or friendly) if each equals to the sum of the aliquot divisors of the other (*aliquot divisors means all the divisors excluding the number itself*).

For example aliquot divisors of number 220 are **1,2,4,5,10,11,20,22,44,55 and 110**. The aliquot divisors of number 284 are **1,2,4,71 and 142**.

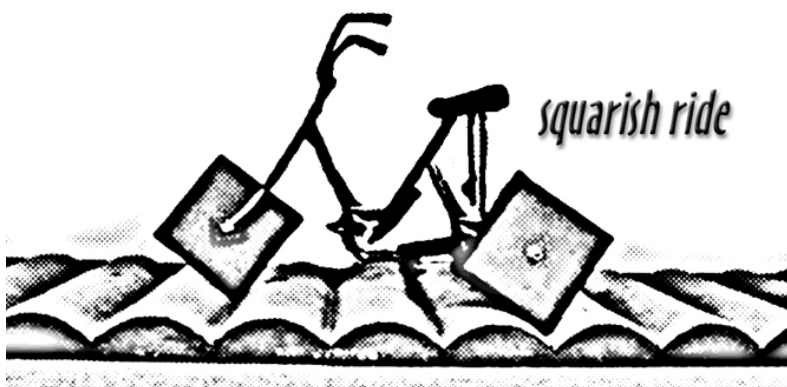
$$\begin{aligned} 1+2+4+5+10+11+20+22+44+55+110 &= 284 \\ 1+2+4+71+142 &= 220 \end{aligned}$$

The smallest amicable pair (220, 284) is known from antiquity and so much significance was attached to it that the possessor of one was assured of close friendship with the possessor of the other number of the pair and so much so some marriages have been made in the past on the basis of amicable numbers. Just find how 1210 and 1184 are amicable.

It was not until 1636 that the great Pierre de Fermat discovered another pair of amicable numbers (17296, 18416).

Chapter 19 BICYCLE WITH SQUARE WHEELS

“ A bridge [...] is a very special thing. Haven't you seen how delicate they are in relation to their size? They soar like birds; they extend and embody our finest efforts; and they utilize the curve of heaven. When a catenary of steel a mile long is hung in the clear over a river, believe me, God knows. [...] the catenary, this marvelous graceful thing, this joy of physics, this perfect balance between rebellion and obedience, is God's own signature on earth. I think it pleases Him to see them raised.” **Mark Helprin** - *Winter's Tale*



Bicycle with square wheels

Suppose your bicycle had wheels, which were square, and not round could you ever roll smoothly without jerks. Strange question, but there is a special kind of track on which a square wheel can move smoothly *without any jerks*. It turns out that a polygon, such as the square above,

can "roll" smoothly on a track made of segments of catenaries.

Catenary is a word that most of us may not have heard unless we had anything to do with transmission lines and railways. The name *catenary* is associated with the curve because it describes the shape formed by a chain or rope freely suspended by its endpoints.

Catenary the name for the curve formed by a hanging rope is actually from the Latin root, *catenareus* , for chain. The word was developed in correspondence between Leibniz and Huygens around 1690, but there seems to be some disagreement about who used the term first.

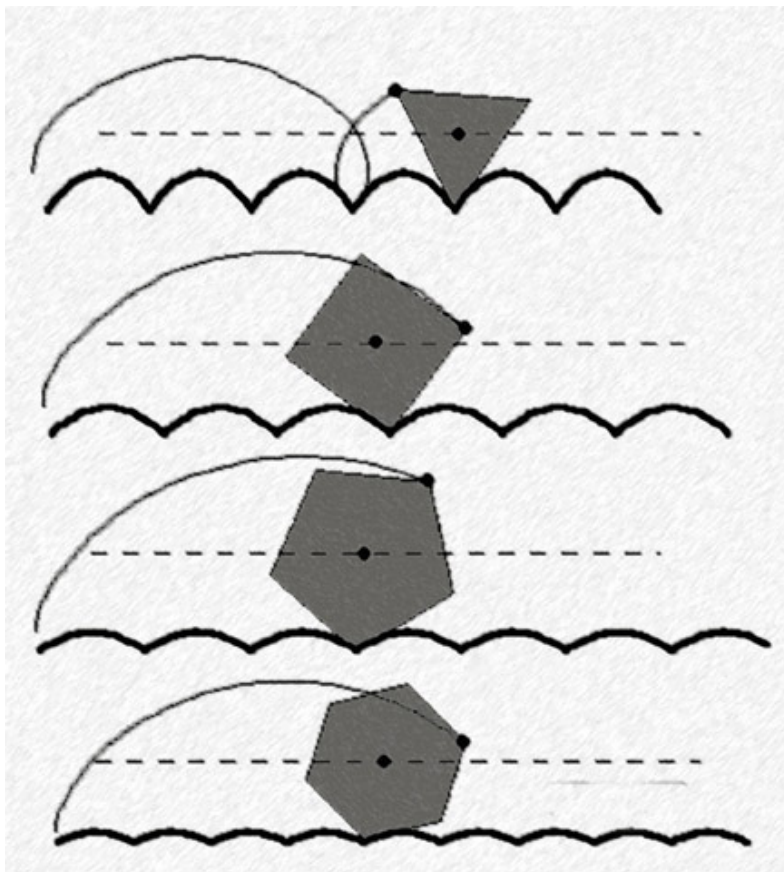
Galileo had believed that a freely hung rope formed a parabola, this was disputed by a mathematician named Jungius. Huygens was the first to use the word catenary.

Mathematically speaking the equation for this curve is

$$y = a \cosh(x/a)$$

So tell your friend now that the squarish tired cycle is a mathematical possibility, only the track laying will be nightmare for the civil engineer. Several types of polygons can be rolled on tracks with segments of catenary in a jerk free fashion. The following diagram depicts this. A special calculation has to be made for no of segments and their spacing.

Remember the circle is a polygon with infinite sides, so a straight track may also be called a catenary track with infinite segments.



(Observe that the center point of each polygon is moving in a straight line)

Chapter 20 THE ETERNAL SEARCH FOR PRIME

WILL THIS EVER END, no it cannot because the worst fear came true when someone declared there are infinite primes. The computers are getting speedier, thus assisting the prime algorithm in searching, verifying and authenticating the *primality* of the number.

After all what is so fascinating about primes? All over the world for centuries together amateurs, serious mathematicians are all so involved in finding primes. There is stiff competition. The internet has a site belonging to GIMPS, *Great Internet Mersenne Prime Search*, which is dedicated to the search of prime numbers.

The largest known prime, as of February 2006, is $2^{30402457} - 1$ (this number is 9,152,052 digits long); it is the *43rd known Mersenne prime*. This number is designated as **M30402457**. It was found on December 15, 2005 by Curtis Cooper and Steven Boone, professors at Central Missouri State University and members of group with GIMPS.

The next largest known prime is $2^{25964951} - 1$ (this number is 7,816,230 digits long); it is the *42nd known Mersenne prime*. **M25964951** was found on February 18, 2005 by Martin Nowak (also a member of GIMPS).

The reader must be wondering on the designation MERSENNE. First for readers who may want to know, what is prime number and why is it so important?

A *prime number* is a natural number greater than 1 that can be divided without any remainder only by 1 and itself. Thus the first few prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...

A prime number of the shape $2^n - 1$ (i.e., one unit less than a power of 2) is known as a *Mersenne prime*. These are named after Marin Mersenne (1588-1648), a French scientist and mathematician. In 1644, Mersenne proposed a tentative list of the powers of 2, which follow a prime number. Following is the list of exponents, n , which are the 43 Mersenne primes known till February 2006.

**2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607,
1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941,
11213, 19937, 21701, 23209, 44497, 86243, 110503,
132049, 216091, 756839, 859433, 1257787, 1398269,
2976221, 3021377, 6972593, 13466917, 20996011,
24036583, 25964951, 30402457**

What is so fascinating about primes? I could not really find an answer despite consulting several mathematicians, but yes some kind of divine experience is associated with primes. Paul Erdos one of pioneering physicists said "*God may not play dice with the universe, but something strange is going on with the prime numbers.*" Someone once said the search for prime is like landing on the moon. It may have no real significance, but the offshoots of the technology are of great utility to mankind. In the tradition of search for prime some of the giants such as Euclid, Euler and Fermat, left in their wake some of the greatest

theorems of elementary number theory. Much of elementary number theory was developed while deciding how to handle large numbers, how to characterize their factors and discover those which are prime.

People love to collect beautiful and antique items. The maths lover would like to attach one's name to a prime number. Their greatest contribution is to the curiosity and spirit of humankind. This search also has big money involved like there are prizes for the first prover ten-million digit prime (\$100000), the first hundred-million digit prime (\$150000), and the first billion digit prime (\$250000).

Rise up to the occasion log on to www.mersenne.org.

Last word, A *prime twin* is a pair of primes that differ by 2. Examples for prime twins are: (3,5), (11,13), and (1000000007,1000000009). The largest known prime twin is

$$242206083 \times 2^{38880} \pm 1$$

Each member of this twin comprises 11,713 decimal digits!

Chapter 21 PERFECT NUMBERS

Ancient Greeks visualized perfection in 6, they called it a PERFECT number. “Six” pertains to the man, God created the world in Six Days.

A perfect number mathematically is an integer, which is the sum of its aliquot divisors (remember the Chapter on amicable numbers, *aliquot divisors means all the divisors excluding the number itself*).

Aliquot divisors of 6 are 1,2,3 and $1+2+3=6$, thus it is a perfect number. The Ancient Greeks knew only of the first four perfect numbers.

$$6 = 1 + 2 + 3$$

$$28 = 1 + 2 + 4 + 7 + 14,$$

$$496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248$$

$$8128 = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 \\ + 508 + 1016 + 2032 + 4064$$

Euclid discovered that the first four perfect numbers are generated by the formula $2^{n-1}(2^n - 1)$.

$$\text{for } n = 2: 2^1(2^2 - 1) = 6$$

$$\text{for } n = 3: 2^2(2^3 - 1) = 28$$

$$\text{for } n = 5: 2^4(2^5 - 1) = 496$$

$$\text{for } n = 7: 2^6(2^7 - 1) = 8128$$

Euclid proved that the formula $2^{n-1}(2^n - 1)$ gives an even perfect number whenever $2^n - 1$ is prime. Do you see a relationship, the Mersenne prime comes into play. In order for $2^n - 1$ to be prime, it is necessary that n should be prime.

Something very unique about the perfect number is that **it is the sum of all natural numbers up to $2^n - 1$** . This follows from the general formula stating that the sum of the first m positive integers equals $(m^2 + m)/2$. Furthermore, any even perfect number except the first one is the sum of the first $2^{(n-1)/2}$ odd cubes.

$$6 = 2^1(2^2-1) = 1+2+3$$

$$28 = 2^2(2^3-1) = 1+2+3+4+5+6+7 = 1^3+3^3$$

$$496 = 2^4(2^5-1) = 1+2+3+.....+29+30+31 = 1^3+3^3+5^3+7^3$$

$$8128 = 2^6(2^7-1) = 1+2+3+...+125+126+127 = 1^3+3^3+5^3+7^3+9^3+11^3+13^3+15^3$$

Just one more thing about perfect numbers, Reciprocals of the divisors of a perfect number add up to 2.

For 6, we have

$$1/6 + 1/3 + 1/2 + 1/1 = 2$$

For 28, we have

$$1/28 + 1/14 + 1/7 + 1/4 + 1/2 + 1/1 = 2$$

Chapter 22 NUMBERS IN BIBLE

I had heard that worship of GOD was the prime motivator for creativity, even the most brilliant of scientists believed in the Supreme Being. Not to say they were religious in the “unreal” sense believing in mindless rituals and hours of praying in the aisle. The Church and Bible fascinated me for its grandeur in building and spirit. I read the book from a mathematician’s point of view. I was trying to find some patterns in all the numbers used in Bible.

The number “one” signifies absolute singleness.

The number “two” indicates witness and support. The Ten Commandments were written on two stones. Jesus’ disciples were sent out in twos.

The number “three” is mentioned numerous times in the Bible. It is the number of unity, of accomplishment, and of the universe. The human race is traced to Noah’s three sons. Jesus’ earthly ministry lasted three years; he rose from the dead on the third day; and the Trinity is three Divine Persons in one God.

“Six” is the number pertaining to man. *The world was created in six days.* Israel marched around Jericho six times.

Saint Augustine (354-430) writes in his famous text **The City of God**,

Six is a number perfect in itself, and not because God created all things in six days; rather, the converse is true. God created all things in six days because the number is perfect...

“Eight” is the new beginning number. Eight were saved from the flood. Circumcision was to be performed on the eighth day.

The damned Triplicate 666

666 is the number of the Satan. It is the damned triplicate. 666 represent humankind in general because of the special significance that the number has in the Bible. Six is known as an "imperfect number" (*confused just a lines before it was called the perfect number, that's mythology!*) because it is one short of seven, the "perfect number" (seven days in the week, seven tongues of flame, seven spiritual gifts...). 666 has three sixes and three is the number of the Trinity, so 666 is seen as extremely imperfect. Therefore, 666 represents imperfect man, while 777 represents God.

Some interesting properties of this number are

$$666 = 2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2$$

$$\begin{aligned} 666 &= 1 + 2 + 3 + 4 + 567 + 89 \\ &= 123 + 456 + 78 + 9 = 9 + 87 + 6 + 543 + 21 \end{aligned}$$

three different ways by using 1 to 9 that too sequentially!

$$666 = (61 + 61 + 61) + (63 + 63 + 63)$$

Incredibly, the number 666 is equal to the sum of the digits of its 47th power, and is also equal to the sum of the digits of its 51st power. That is,

$$666^{47} =$$

$$\begin{array}{r} 504996968442079675317314879840556477294151 \\ 629526540818811763266893654044661603306865 \\ 302888989271885967029756328621959466590473 \\ 3945856 \end{array}$$

$$666^{51} =$$

$$\begin{array}{r} 993540757591385940334263511341295980723858 \\ 637469431008997120691313460713282967582530 \\ 234558214918480960748972838900637634215694 \\ 097683599029436416 \end{array}$$

In fact, 666 is the only integer greater than one with this property. (Also, note that from the two powers, 47 and 51, we get $(4+7)(5+1) = 66$.)

Strange ways!

$$\begin{array}{r} 1^3 = 1^2 \\ 1^3 + 2^3 = (1 + 2)^2 \\ 1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2 \\ 1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2 \end{array}$$

Chapter 23 PALINDROMES

Palindromes – words which spell same backward and forward have been a source of attraction for linguists, in mathematics palindromic numbers have also fascinated the number maniac. Some striking patterns can be conceived.

I got this from Martin Gardner's book.

Think of a two-digit number. Reverse the digits (05 for 50) and add this number to the selected one. Repeat the steps till you get a palindrome.

Try 49

$$49 + 94 = 143$$

Repeat the steps, $143 + 341 = 484$ a palindrome !

This is true for almost all numbers.

Warning use a calculator as some numbers get very large.

Just try 89, can you even guess what number is generated?

8813200023188

Chapter 24 WEIGHTS AND LESSER WEIGHTS

The Problem of weighing using logic has thrilled several commoners and had potential to irritate the mathematician for want of proof. Some very interesting problems and solutions will follow, it is advised that the reader stop and think, preferably with a rough paper and pen.

100 KGS WITH FIVE STONES

What should be weight of each stone so that the grocer can weigh 1 to 100 kgs using only five stones?

A variant of the problem what should be minimum number of stones and what weight, so that he is able to weigh 1 to 100 using the weights on only one side of the balance ?

FIVE BALLS

Mr Deep has a set of five balls, identical in touch and looks. But none of them have similar weights. He has to arrange the balls in order of weight from the heaviest to lightest. At his disposal is a balance and no weighing blocks. What is the minimum number of weighing required for this ?

Hint: Seven

MEASURE 9 GRAMS

In a bag there are 24 grams of nails, can you measure 9 grams using a balance with two pans.

101 COUNTERFEIT COINS

Mr. Pal has 101 coins. He is told that 50 of them are counterfeit and differ by 1 gram from genuine coins. He has a scale with two pans, which can show the difference in weight between the two sets of objects placed in each pan.

He chooses one coin and in one weighing wants to know whether it is counterfeit.

LIGHTER GOLF BALL

The Golfer bought ten bags of golf balls. But it was later known that one of the bags contained balls lighter by 1 gram. Using only one weighing on a digital balance, find out the culprit bag. The weight of each ball is 10 g.

VARIANT TO THE GOLFER'S PROBLEM

The Golfer used to buy golf balls in packs of six. He suspected that the dealer was cheating him by including one box of substandard balls in set of ten boxes. The substandard balls were same in size and appearance only a gram lighter.

Once after receiving a set of 10 boxes with six balls in each box, he prepared to find out the faulty box. He only had a pair of scales and set of weights. How could he do it in just one chance? Well he did not know the weight of the golf ball.

The following is by far the best logical problem I have ever solved.

13 BALL PROBLEM

You have a balance with two pans but no weights. There are 13 balls, which are identical in size and shape. But one of them is defective, **it is lighter or heavier than the rest.** *(please note this carefully otherwise like most you would shout Eureka, being miles away from the solution)*

Problem is to find out the odd ball in just three weighing.

Hint: Label the balls 1 to 13 and form groups of four. This is not the only way but certainly the easiest.

14 COIN PROBLEM

In a set of 14 coins exactly 7 coins are counterfeit and weigh less than the genuine coins. How can the counterfeit and genuine coins be found out in only three weighing?

This is left unsolved, but it definitely has a solution, unknown to author.

Discussion and Solutions

100 kg with five stones

Using both sides of the balance, he needs just **1, 3, 9, 27 and 81 kg** weights. This is a special property of base 3 numbers.

Suppose he had to weigh 25 kgs, the 27 kg and 1 kg block would be placed at one side and 3 kg block along with the item on the other side. Thus he has **$27+1-3=25$ kg**.

Similarly for 100 kgs, he has to place 81, 27 and 1 on one side and 9 on the other side, to have **$81+27+1=109$ and $109-9=100$ kg!**

The Variant to the problem wants us to use only one side of the pan here base 2 property is used, the least number of weights required are 7 i.e. **1, 2, 4, 8, 16, 32 and 64**.

Suppose you were to weigh 25 kg, take 16, 8 and 1 kg weights.

To get 100 kg, take 64, 32 and 4. You can calculate similarly for all units in between 1 to 100.

Five balls

1. Weigh any two balls label them H and L
2. Weigh any two from remaining three balls, label h and l

3. Now weigh H with h , whichever is heavier redesignate as H and h . Put L aside and have H, h, l in order.
4. In 4 & 5 weighing it is possible to place the fifth ball in order among H, h, l
5. In the 6 & 7 weighing i.e. two weighing regardless of the placement of fifth ball, the ball designated L can be placed using the fact that L is lighter than H .

Note: steps 4 and 5 have been purposefully kept imprecise for you to appreciate the logic. Try it out.

Measure 9 grams

Use the balance to divide 24 grams into 12 grams each. Then divide 12 grams into two equal parts of 6 grams. Keep one part separately.

Now divide 6 grams into two parts of 3 grams each. Take one part of this and put the 6 gram heap into one side of pans. Now you can measure 9 grams. Simple!

101 Counterfeit coins

Lay aside the chosen coin. Divide the coins in piles of 50 coins each. If the chosen coin is genuine then difference in weights of the two piles will be even, otherwise odd. The logic is too good to be explained, *try it yourself*.

Lighter Golf Ball

Take 1 ball from first bag, 2 from second and so on 10 balls from tenth bag.

The total number of balls is 55, correct weight should be 550 g.

After getting weight of the set of balls it can easily be identified which bag is defective.

For example 545 g would mean 5 balls are defective and five balls were taken from the fifth bag.

Variant to the Golfer's problem

Label the boxes A to J.

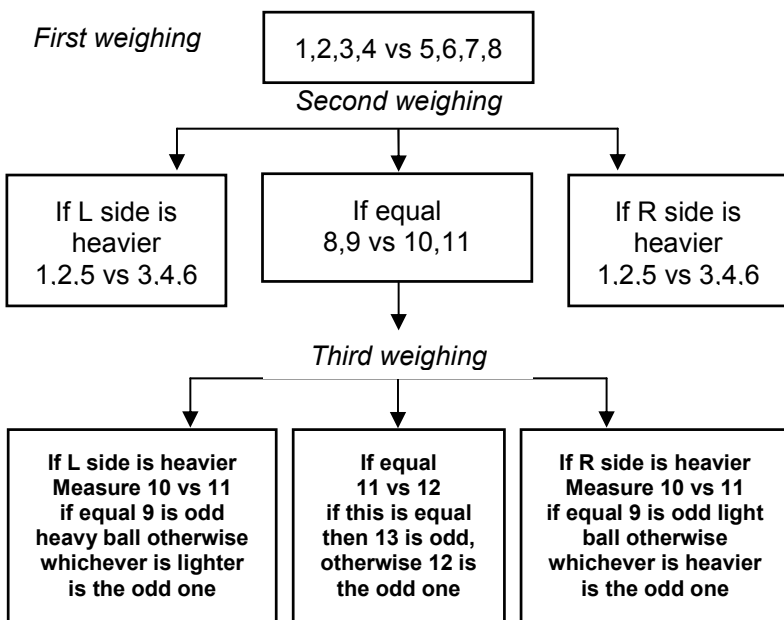
Now he should take 1 ball from A, 2 from B, 3 from C, 4 from D, 5 from E and balance them against 1 from F, 2 from G, 3 from H, 4 from I and 5 from J. Now using the weights he should balance the scales. The number of grams required and side tell the faulty box.

For example, if 4 grams were needed on the left pan, it would mean 4 balls are lighter and 4 balls had been taken from box D. If H had been the faulty box, then 3 grams would be required on the right side.

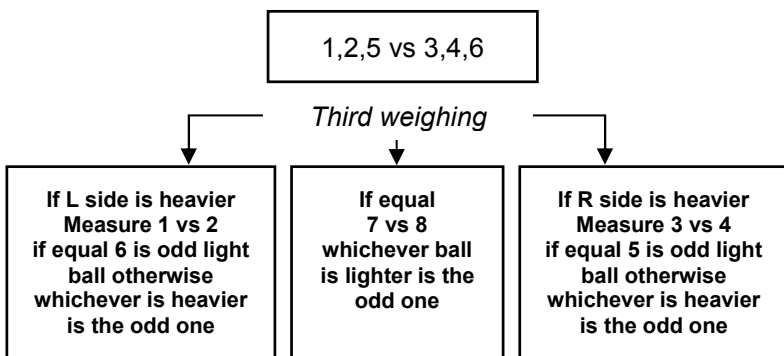
13 Ball Problem

THIS THE BEST LOGICAL PROBLEM YOU CAN SOLVE, so try once more before looking at the solution.

First number the balls from 1 to 13 and group them as 1,2,3,4; 5,6,7,8; 9,10,11,12 and keep 13 aside.



Now we shall consider L side heavier after first weighing



The R side heavier case after first weighing can be solved using the same logical sequence of weighing

Chapter 25 BEST CURRENCY SYSTEM

Have you ever wondered on why we have currency note designation as 1,2,5,10,20,50? Is it mathematically the best combination or is it for convenience as adding up these numbers together is easier.

I made a small calculation that if at one time I had to produce all denominations from Rs 1 to 100, what would be the total number of notes and of what denomination required for me to keep in my pocket,

The answer is one note each of Rs 1,5,10,50,100 and two notes each of Rs 2 and 20. That is 9 notes in all.

But a small observation shows that there is a way to have denominations such that we are required to *carry only one note of each type*.

Can you try it out ? Stop reading for some time ponder upon this very interesting calculation.

The denominations are 1, 2, 4, 8,16,32 and 64, seven notes in all.

See the table below.

This shows the total no of notes required to have Rs 1 to 100 with both systems. An exercise for you is how?

Currency	New	Old
1	1	1
2	1	1
3	2	2
4	1	2
5	2	1
6	2	2
7	3	2
8	1	3
9	2	3
10	2	1
11	3	2
12	2	2
13	3	3
14	3	3
15	4	2
16	1	3
17	2	3
18	2	4
19	3	4
20	2	1
21	3	2
22	3	2
23	4	3
24	2	3
25	3	2
26	3	3
27	4	3
28	3	4
29	4	4
30	4	2
31	5	3
32	1	3

33	2	4
34	2	4
35	3	3
36	3	4
37	4	4
38	3	5
39	4	5
40	2	2
41	3	3
42	3	3
43	4	4
44	3	4
45	4	3
46	4	4
47	5	4
48	2	5
49	3	5
50	3	1
51	4	2
52	3	2
53	4	3
54	4	3
55	5	2
56	3	3
57	4	3
58	4	4
59	5	4
60	4	2
61	5	3
62	5	3
63	6	4
64	1	4
65	2	3

66	2	4
67	3	4
68	2	5
69	3	5
70	3	2
71	4	3
72	2	3
73	3	4
74	3	4
75	4	3
76	3	4
77	4	4
78	4	5
79	5	5
80	2	3
81	3	4
82	3	4
83	4	5

84	3	5
85	4	4
86	4	5
87	5	5
88	3	6
89	4	6
90	4	3
91	5	4
92	4	4
93	5	5
94	5	5
95	6	4
96	2	5
97	3	5
98	3	6
99	3	6
100	3	1

The total number of notes required with the present system is 341 and with the new Mathematically correct system it is 320.

Try Rs 50 with new system,

$50 = 32 + 16 + 2$, it is a bit inconvenient, though.

All this is to be taken as mathematical recreation. The present system is the best!!

Again my obsession leads me to a problem. Is there no simpler way with lesser denominations to have, say only four currency notes?

Various combinations were tried but finally using computer programme the four denominations using least number of notes to produce Rs 1 to 100 was found with **1,5,18,25** – here there is no precondition of using them only once. Try figuring it out? But I would tell you the total number of notes required 393!

Chapter 26 WHY DECIMAL SYSTEM

There are 10 fingers. The fingers are the first things we use to calculate or count. Thus there is no doubt that base 10 is the most convenient. True, but then why so many other base systems across ages, cultures and geographical locations. Good question.

The origins of our modern decimal, or base-10, number system can be traced to ancient Egyptian, Babylonian (Sumerian), and Chinese roots. The real credit for the base-10 system with a symbol for zero goes to the Hindu mathematicians of the fifth and sixth centuries. After their discovery of the system it was transmitted to mathematicians in the Islamic world who developed it to include decimal fractions during the period from the 9th to 11th centuries AD. The introduction of this system in the West took place with the translation of the treatise of *Muhammad ibn Mūsā al-Khwārizmī* (c.780–850) in the twelfth century.

The art of reckoning (*ganita*) was held in the highest esteem by the ancient Indians. They used symbols for marks or divisions (*ankas*) which are the ancestors of modern decimal digits (1, 2, 3, 4, 5, 6, 7, 8, 9). The introduction of a tenth symbol for zero (0) paved the way to *positional system* of decimal numeration. The Sanskrit name for zero is *shoonya* ("void", "nothingness" or "emptiness").

The need for *zero* was circumvented in India, as it had been in the everywhere else. Instead, the ancient Indians used different Sanskrit words for *all* the successive powers of ten.

The original scheme called for first naming the largest possible power of ten which could go into a given integer, along with a *nonzero* anka (from 1 to 9) stating how many times it could do so (the rest of the integer, if any, being named according to the same recursive scheme).

About 2000 years ago, it occurred to an extraordinary Indian that the powers of ten need not be mentioned at all, *provided* a symbol is unambiguously given for each power of ten. This symbol took the form of a small circle with a center dot. Thus zero was born, the center dot has been dropped in the modern "0". Thus was born the modern decimal system.

The oldest surviving reference to this modern decimal system is a sacred text called *Agni Purana*. The new system was used (with multiplication tables) shortly thereafter in Roman Syria [modern Jordan] by Nicomachus of Gerasa (AD 60-120) in his famous *Arithmetike Eisagoge* ("Introduction to Arithmetic"). Arithmetic as a separate field of study from geometry was established through this work.

Chapter 27 HCF AND LCM

The concept of LCM and HCF is probably the most confusing mathematical concept. To all readers who remember anything in high school maths, this would be last thing they can recall correctly. Try to calculate LCM for 20 and 48.

Let us make it simple. LCM – *least common multiple*, it is the least multiple which can be obtained after multiplying any number in the set with some other number.

For example,

20, 24

Factors are

$$\mathbf{20 = 2 \times 2 \times 5}$$

$$\mathbf{24 = 2 \times 2 \times 2 \times 3}$$

so the LCM is 60 as it is the least number which is a multiple of 20 and 24.

Now coming to HCF, this is the Highest common factor for a set of numbers, in other words it is highest such number (divisor) which can divide all numbers without a remainder.

Let us consider 20 and 24, they are both divisible by 2 and 4 but 4 is the highest common divisor. Taking a more complex example,

48,60,120

Here 2,3,4,6,12 all divide the three numbers but 12 is the highest among them.

Interestingly

$\text{HCF} \times \text{LCM} = \text{product of the numbers.}$

How are these concepts useful in arithmetic and algebra?
Consider the following problems

1. The length and breadth of a room are 3 metres and 2 metres respectively. What will be the size of the square tile if its floor is to be covered fully with minimum number of square tiles of the same size without breaking them?
2. There are two sections A and B of a certain Class in a school. The children of Section A organise a quiz competition after every 15 days and those of Section B after every 20 days. If both sections organise it together on the first day of the session, after how many days will they organise it together again?

The concepts of HCF and LCM are used here.

Problem 1, here the size of the tile should be the highest number that divides completely both 3 and 2, i.e. the HCF

of 3 and 2, which is 1. So the size of the square tile should be 1m by 1m.

Suppose the room was of 6 m by 4 m, the square tiles of 2m by 2m would completely cover the room without breaking any tile as HCF of 6 and 4 is 2.

Problem 2, in this case the number of days would be the least number that is a multiple of both 15 and 20, i.e. LCM of 15 and 20.

$$15 = 5 \times 3$$

$$20 = 4 \times 5, \text{ thus } \text{LCM} = 3 \times 4 \times 5 = 60$$

So after exactly 60 days they would be organising the event together again.

Chapter 28 POWER OF INDICES, REACHING THE MOON AND INVENTOR OF CHESS

Some of the most intriguing problems and facts came after we learnt the power of “powers”, i.e. indices, laws of exponents.

If you were to reach the moon by folding a paper, how many times would you fold the paper of say 0.1 mm thickness?

The Moon is roughly 3.85×10^8 m from earth, not much really just 3,85,000 kms.

So per fold the thickness of paper doubles i.e. after the first fold it becomes

$$0.1 \times 2 = 0.2 \text{ mm}$$

Distance to moon is 3,85,00,00,00,000 mm

Let n be the number of folds then

$$2^n \times 0.1 = 3,85,00,00,00,000 \text{ mm}$$

n calculated using logarithm is approximately 42.

Unbelievable, only 42 folds are required for reaching the moon.



An exercise for you.

The Sun is about 150000000 kms from earth, how many fold will be required for reaching the sun?

INVENTOR OF CHESS

There was a very intelligent man who invented the game of chess. The ruler was so amused that he granted him a wish to ask for anything. Remember he was very intelligent. He stated his wish thus

"I am simple man, not interested in glory, mansion or lots of money. I just need to feed myself. So, my dear Lord please grant me my wish to have grains of wheat which can fit on the chess board."

The ruler was taken aback, "That's all, come on you are insulting me"

"No, my Lord I have not completed, but please promise you will honour your commitment."

“Yes, of course”

“Ok, place one grain on the first square on the board, double the quantity on the second and so on till the 64th square. I just want the all these grains.”

The ruler was amused but but he did not keep his commitment. He couldn't have.

STOP, do not read further. Try figuring it out why? In case you are bogged down, read ahead.

The number of grains on the 64th square is 2^{63} (2 raised to the 63rd power).

The total number of grains on the board is $2^{64}-1$.

These facts can be easily deduced by considering just the first few squares, and generalizing your findings. A proof can be done using mathematical induction, or geometric series, or binary arithmetic.

$$2^{64}-1 = 18,446,744,073,709,551,615$$

That happens to be much more wheat than exists in the whole world. In fact, that amount of wheat would probably just fit in a warehouse 40 kilometers long, 40 kilometers wide, and 300 meters high.

Chapter 29 INSTANT CUBE ROOT-MATHEMAGIC

Cube Roots up to 6 Digits

What is the cube root of 262144?

The cube root of a 6-digit number has to be less than 100, so you could consider memorising the first 100 cubes. Here is superb way to dazzle your friends with your astonishing calculation ability.

Divide the number in two parts, put a separator after the first three digits 262,144.

The left part 262 is between $216 = 6^3$ and $343 = 7^3$. This means 262144 lies somewhere between $216000 = 60^3$ and $343000 = 70^3$. Hence the answer is surely between 60 and 70.

Something very special about cubes of single digits is that the last digit is unique. To understand let us see the following table, the second row is cube of respective first row number.

1	2	3	4	5	6	7	8	9
1	8	27	64	125	216	343	512	729

So all you have to know rather remember very definitely is these cubes and the associated last digit which is unique. For 1,4,5,6 and 9 it is the same. For 3,7 and 2,8 last digits are reverse of the set. Not a tough task at all after a few minutes of practice.

Applying this to our cube 262144 in just 15 seconds the answer 64 will emerge shocking all your friends.

Lets try finding cube root of 571787.

*Answer in 10 seconds now, **83** »*

as 571 is between 512 and 729 i.e. first digit of cube root is 8.

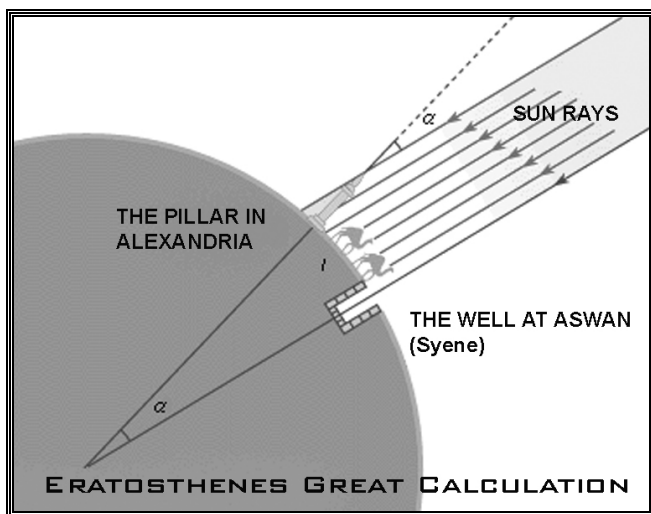
Since 7 is the last digit associated with cube of 3, therefore we have 3.

There are very interesting ways to find cube roots using unconventional methods, but this one is perfect for *showing off*, as it involves absolutely no calculations and is actually mental.

Chapter 30 CALCULATING SIZE OF EARTH – TRIGONOMETRICAL POTENTIALITIES

Since its earliest days, geometry has been applied to practical problems of measurement—whether to find the height of a pyramid, or the area of a field, or the size of the earth. “Geometry” derives from the Greek *geo* (earth) and *metron* (to measure). But the ambition of the early Greek scientists went even farther: using simple geometry and later trigonometry, they attempted to estimate the size of the *universe*.

In the year 240 b.c. Eratosthenes, a famous Greek scientist achieved the feat for which he is chiefly remembered, *computing the size of the earth*.



It was known that at noon on the day of the summer solstice (the longest day of the year), the sun's rays directly illuminated the bottom of a deep well in the town of Syene (now Aswan) in Upper Egypt. The sun was exactly overhead at noon as Aswan lies on the Tropic of Cancer. The shadow of a vertical rod in Alexandria, due north of Syene, showed that the sun was about 7.2 degrees from the Zenith, this is about one-fiftieth of a full circle (360°).

Eratosthenes very rightly assumed that the sun is so far away from the earth that its rays reach us practically parallel, hence the difference in the sun's elevation as seen from the two locations must be due the sphericity of the earth. Since the distance between Alexandria and Syene was 5,000 stadia (as measured by the time it took the king's messengers to run between the two cities), the circumference of the earth must be fifty times this distance, or 250,000 stadia.

The exact length of the stadium, the geographical distance unit in the Greek era, is not known; estimates vary from 607 to 738 feet, the smaller figure referring to the Roman stadium of later use. The circumference of the earth as found by Eratosthenes is therefore between 29,000 and 35,000 miles. The correct value is amazing close 24,818 miles for the polar circumference and 24,902 miles for the equatorial. Eratosthenes used the science of geometry in its literal sense: *to measure the earth*.

Barnabas Hughes, in his *Introduction to Regiomontanus' On Triangles* said,

It is quite difficult to describe with certainty the beginning of trigonometry. In general, one may say that the emphasis was placed first on astronomy, then shifted to spherical trigonometry, and finally moved on to plane trigonometry.

Now a basic query, how did this word *sine* originate, there are several stories told in different ways, but what is found to be most authentic was an early Hindu work on astronomy, the *Surya Siddhanta* gives a table of half-chords based on Ptolemy's table. But the first work to refer explicitly to the sine as a function of an angle is the *Aryabhatiya* of *Aryabhata* (ca. 510), considered the earliest Hindu treatise on pure mathematics. In this work *Aryabhata* (also known as *Aryabhata the elder*; born 475 or 476, died ca. 550)² uses the word *ardha-jya* for the half-chord which is shortened to *jya* or *jiva*.

The etymological journey of the modern word “sine” is interesting and starts from here. When the Arabs translated the *Aryabhatiya* into their own language, they retained the word *jiva* without translating its meaning. In Arabic and Hebrew, words consist mostly of consonants, the pronunciation of the missing vowels being understood through common usage. Thus *jiva* could also be pronounced as *jiba* or *jaib*, and *jaib* in Arabic means bosom, fold, or bay.

When the Arabic version was translated into Latin, *jaib* was translated into **sinus**, which means bosom, bay, or curve. Soon the word sinus—or sine in its English version—became common in mathematical texts throughout Europe. The abbreviated notation sin was first used by Edmund

Gunter (1581–1626), an English minister who later became professor of astronomy at Gresham College in London. In 1624 he invented a mechanical device, the “Gunter scale,” for computing with logarithms—a forerunner of the familiar slide rule—and the notation $\sin$ (as well as $\tan$) first appeared in a drawing describing his invention.

The remaining five trigonometric functions have a more recent history. The cosine function, which we regard today as equal in importance to the sine, first arose from the need to compute the sine of the complementary angle. Aryabhata called it *kotijya*. The name ***cosinus*** originated with Edmund Gunter: he wrote *co sinus*, which was modified to ***cosinus*** by John Newton (1622–1678), a teacher and author of mathematics textbooks (he is unrelated to Isaac Newton) in 1658. The abbreviated notation ***cos*** was first used in 1674 by Sir Jonas Moore (1617–1679), an English mathematician and surveyor.

The functions secant and cosecant came into being even later. The word “tangent” comes from the Latin ***tangere***, to touch. Its association with the tangent function may have come from the fact that the tangent to a circle is related to this function on the unit circle.

The science of trigonometry was in a sense a precursor of the telescope. It brought faraway objects within the compass of measurement and first made it possible for man to penetrate in a quantitative manner the far reaches of space.

Stanley L. Jaki, *The Relevance of Physics* (1966).

Chapter 31 SEQUENCES AND SERIES

Carl Friedrich Gauss, the great German mathematician was in elementary school when his teacher asked the class to find the sum of first 100 natural numbers. While the rest of the class was struggling with the problem, Gauss had the answer within no time.

The method used by Gauss to find the sum was the formula for calculating sum of n terms of an A.P. In other words, the sum of the first n terms in an A.P. is n times the average of the first and the last term in the A.P.

A.P., what is this new term? ***Arithmetic Progression***. It is a series of numbers such that there is constant difference between successive terms. For example, 2,5,8,... here the difference between first and second, second and third terms is 3.

Sum of A.P. series is given by formula,

$$S_n = \frac{n \times (2a + (n-1).d)}{2}, \text{ where}$$

S_n = sum of series

a = first term of the series

d = constant difference

n = number of terms

There is another very useful and interesting series of numbers called Geometric Progression or G.P. in short. The successive terms in this case have a constant ratio i.e. ratio

of second term with first term is equal to ratio of last term with second last term. For example, 2,6,18,54,... here ratio of successive terms is 3.

The sum of GP is given by the formula

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ for } r < 1 \text{ and } \frac{a(r^n-1)}{r-1}, \text{ for } r > 1$$

Where,

S_n = sum of series

a = first term of the series

r = constant ratio

n = number of terms

Using these formulae some very intuitive solutions can be found for series of numbers.

For example, can you find the sum of the series upto n terms,

9, 99, 999, 9999, 99999,

At the first glance there seems to be no easy way, but wait think a while and then proceed.

This is not a GP, but it can be converted to one like this

10, 100-1, 1000-1, 10000-1,..... or

10-1, 10²-1, 10³-1, 10⁴-1,, 10ⁿ-1

Thus, $S_n = (10 + 10^2 + 10^3 + \dots n \text{ terms}) - (1 + 1 + 1 + \dots n \text{ times})$

Using the formula for GP above,

$$S_n = \frac{10(10^n - 1)}{10 - 1} - n = \frac{10(10^n - 1)}{9} - n$$

Here r , common ratio = 10 and a , first term = 10

Now can you appreciate the POWER of series, these are used for various applications in physics, chemistry, biology etc.

The recurring decimals can also be expressed as a geometric series. Here is an exciting explanation,

Consider the recurring decimal 0.3333333.....

This can be written as,

$$0.333333.... = 0.3 + 0.03 + 0.003 + 0.0003 + ... \text{upto infinity}.$$

Observe carefully the Right hand side of the equation. This can be perceived as a GP with

first term $a = 0.3$,

common ratio $r = 0.1$ and

number of terms $n = \text{infinity}$

In the formula for $r < 1$ we have, r^n approaching zero as for any number less than one raised to power infinity is zero.

$$S_n = \frac{a(1 - r^n)}{1 - r}, \text{ Here inserting the values we get}$$

Sum = $0.3/(1-0.1) = 1/3$. Great!

Did you know?

$$3^{1/2} \times 3^{1/4} \times 3^{1/8} \times 3^{1/16} \dots = 3,$$

Does it not seem a bit strange? Using the power of GP this can be proven. Before we proceed a small thing about indices.

$$a^b \times a^c \times a^d = a^{b+c+d}$$

$$\text{Now, } 3^{1/2} \times 3^{1/4} \times 3^{1/8} \times 3^{1/16} \dots = 3^{(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots)}$$

$$\text{The series } \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1,$$

As here $a=1/2$, $d=1/2$ and n is infinity.

So the RHS becomes just 3. Try this one.

A climber is 1.6 meter tall. Its height increases by 5 cm in the following year. If next year and so on its height increases by half the amount it increased the previous year, show that it can never be 1.7 meter in height.

Chapter 32 AMAZING INDUCTIVE POWER- MATHEMATICAL INDUCTION

This is probably the most simplistic of mathematical techniques and quite unbelievable in the beginning. Most of us would wonder at the truthfulness of the technique and how could be accepted like that. Let me explain a bit and then see it for yourself.

In drawing scientific conclusions, there are two fundamental processes of reasoning that are commonly employed. One is the process of *deduction*, i.e. the process of reasoning from general to particular and the other is known as the process of *induction* which proceeds from particular to general.

The word induction means the method of reasoning about a general statement from the conclusion of particular cases. Induction begins by observations. We observe and use our intuition to arrive at a tentative conclusion called *conjecture*.

In some cases even if we do not have a counter example, we cannot conclude that a general statement is true simply because it has been found to be true in all its particular cases that have been verified. This raises the question as to how shall we prove a general statement, which is true in some particular cases. Such mathematical statements can be proved by using the method known as mathematical induction, *not proven to be false as yet*.

The Principle of Mathematical Induction

Let $P(n)$ be a statement involving natural number n such that

- (i) $P(1)$ is true,
- (ii) If $P(k)$ is true, then $P(k + 1)$ is also true, where k is a natural number.

Then the statement $P(n)$ is true for every natural number n .

In other words, to prove the statement $P(n)$ to be true for all natural numbers, we must go through two steps. Firstly, we must verify that $P(1)$ is true. Secondly, we must prove that $P(k + 1)$ is true whenever $P(k)$ is true, where k is a natural number.

We now consider some examples.

Can there be induced formula for the sum of first n odd natural numbers?

Rather let us try to prove that the sum of first n odd natural numbers is n^2 .

Let $P(n) = 1 + 3 + 5 + \dots + (2n - 1) = n^2$

$2n - 1$ is the n^{th} odd natural number.

For $n = 1$, $1 = 1^2$

Assuming $P(k)$ is true, i.e. $P(k) = 1 + 3 + 5 + \dots + (2k - 1) = k^2$

Now it shall be proven that $P(k+1)$ is true whenever $P(k)$ is true.

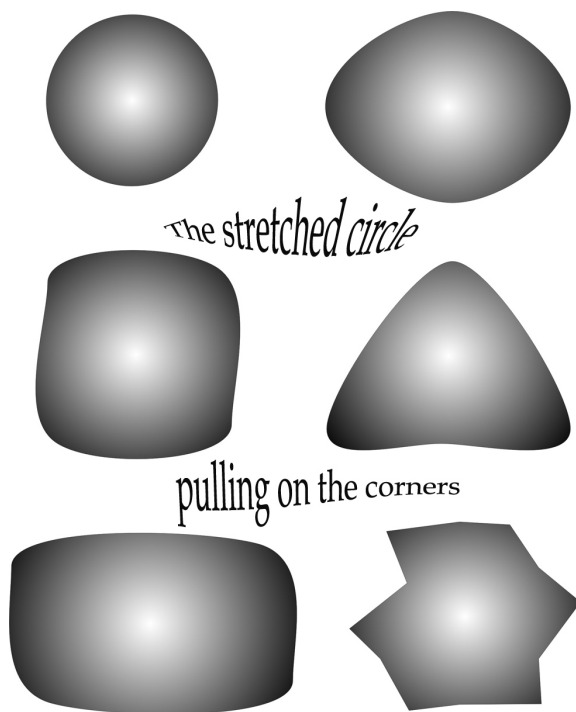
$$\begin{aligned}
 P(k+1) &= \underbrace{1+3+5+\dots+(2k-1)}_{P(k)} + [2(k+1)-1] \\
 &= k^2 + 2k + 2 - 1 = k^2 + 2k + 1 = (k+1)^2
 \end{aligned}$$

Thus $P(k+1)$ is true whenever $P(k)$ is true. Hence by the Principle of Mathematical Induction sum of first n odd natural numbers is n^2 .

Like many of the concepts and methods of mathematics, proof by mathematical induction is not the invention of a individual mathematician. Basically, the principle of mathematical induction was known to the Pythagoreans (600 B.C.). The French mathematician Blaise Pascal is credited with the origin of the principle of mathematical induction. However, the Italian mathematician Francesco Maurolycus had earlier used the principle in his writings. Traces of mathematical induction were found in the writings of the Indian mathematician *Bhaskaracharya* II (1114 – 1185 A.D.). The name induction was perhaps used by the English mathematician John Wallis. Later, the Swiss mathematician James Bernoulli, employed the principle without using the name, to provide a proof of the Binomial Theorem.

Chapter 33 STRANGE GEOMETRY - TOPOLOGY

Draw a circle on a rubber sheet. Now just think what if you stretch only the two sides of the sheet. The circle converts to an ellipse. If the four sides of the sheet are stretched even a square could be made. Thus all kinds of polygon can be made by manipulating the pulls at the corner or sides of the rubber sheet.



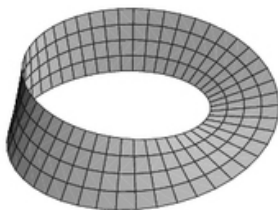
CIRCLE ON RUBBER SHEET

Hence it is concluded that a circle, square, rectangle, triangle, polygons are all TOPOLOGICALLY equivalent.

Topology comes from the Greek root *topos* (place). Before it was used in mathematics, it was applied to the geographic study of a place in relation to its history. It is an offshoot of geometry that originated during the 19th century and that studies those properties an object retains under deformation—specifically, bending, stretching and squeezing, but not breaking or tearing. *Thus, a triangle is topologically equivalent to a circle but not to a straight line segment.*

Similarly, a solid cube made of modeling clay could be deformed into a ball by kneading. It could not, however, be molded into a solid torus (ring) unless a hole were bored through it or two surfaces were joined together. A solid cube is therefore not topologically equivalent to a finger ring.

Probably most of us would have witnessed a very strange looped strip of paper, which when cut through its centre along the length simply becomes a longer loop than getting cut into two parts. The famous MÖBIUS strip. Mathematically it is called a single sided surface.



If an ant were to start moving on a Mobius strip from one point on the surface can you guess where would it reach after completing one round of the strip.

The friction on the pulleys wears the driving belt. If the driving belt is given a 180° twist before the ends are sewn, it will last longer as it will wear equally on both sides, because now it has only *one side*.

Topology involves problems about deformation of elastic bodies and surfaces. The assumption is that the objects considered are made from a very elastic material. Their shape can be changed at will, they can be bent, distorted, stretched and compressed as much as desired. But you can't tear them or glue parts together.



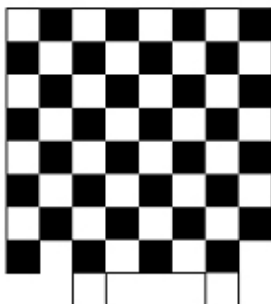
*The elastic body on the left can be deformed to make the one shown on right side or vice versa. **That's rubber sheet geometry!***

Solution next page

	Move one link inside the other
	Now move the inner part as depicted to form another loop
	The upper loop should be pulled
	Pull it a bit more and get a linked torus
	The unlinked torus is converted to linked torus

Chapter 34 SOME DISSECTIONS

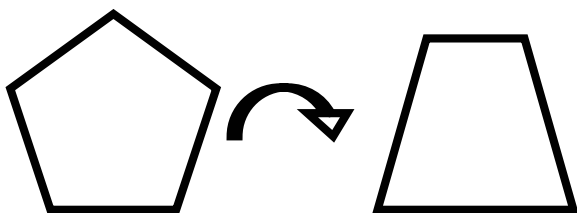
THE IMPERFECT CHESSBOARD



The above chess board is to be divided in two parts so that the parts fit in to make the perfect chess board.

Dissections is perhaps the most recreational aspect of geometry. It has the advantage of being extremely non equation, non formula based using only common sense and lots of logic.

PENTAGON TO TRAPEZIUM

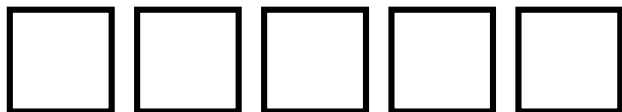


Cut the regular pentagon into four isosceles triangle so that they can be arranged to form a symmetric trapezium.

The next one is different, think differently, laterally!

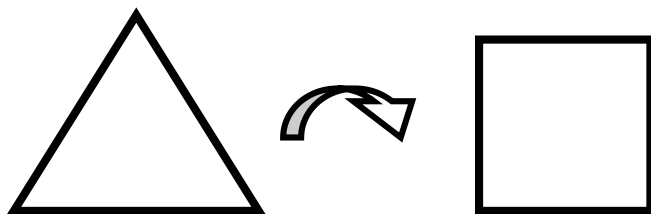
THE SQUARE PROBLEM

There are five identical squares. Make just one straight cut so that they can be rearranged to form a bigger square.



EQUILATERAL DISSECTION

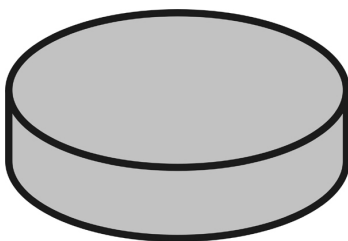
Can you dissect an equilateral triangle in four pieces such that they can be formed into a square? *This one is tough.*



EIGHT PIECES OF CAKE

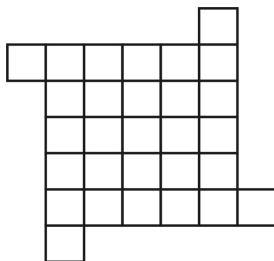
There were eight small children, all of them equally notorious and demanding. Papa got them a cake but now the problem was of dividing the cake into eight exactly equal parts. Papa had time enough to use only three straight cuts.

How could he divide the circular cake into EIGHT equal parts using only THREE straight cuts?



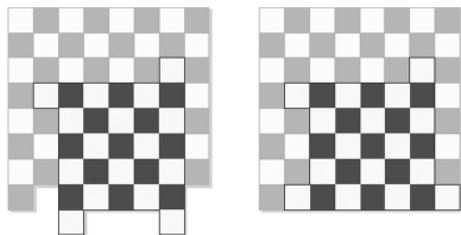
One last problem.

There are 29 units of small squares in the following figure. Your task is to cut the figure into 4 pieces so as to form a square with 29 units. It is not necessary to cut along the lines.



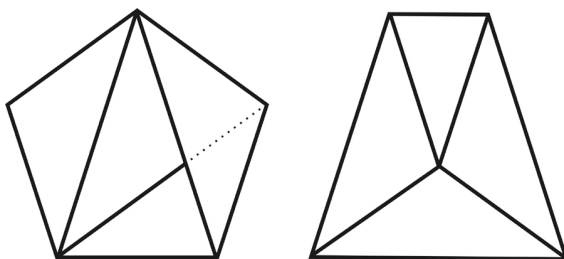
Solutions

THE IMPERFECT CHESSBOARD



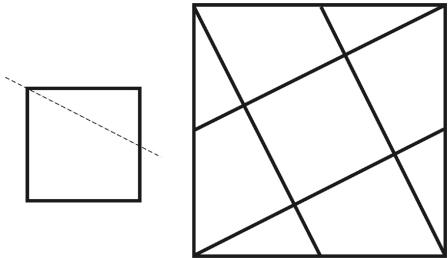
Cut the chessboard as above and rotate the piece by 90°

PENTAGON TO TRAPEZIUM

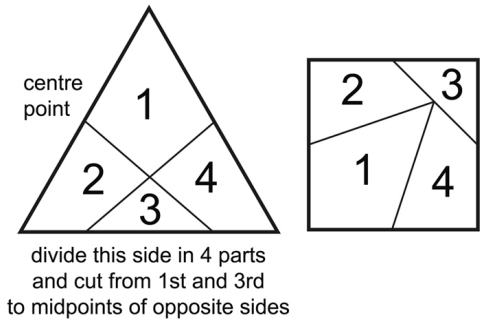


THE SQUARE PROBLEM

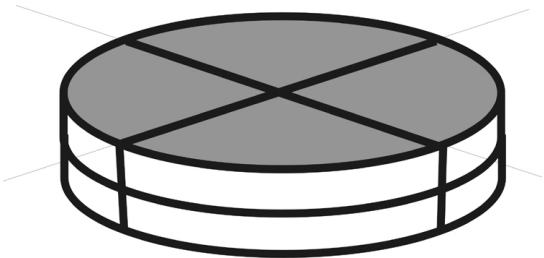
This one is really different. Place four of the pieces one upon the other and make a straight cut passing through one corner and center of the opposite side as shown. Now place the pieces to form a square. Great.



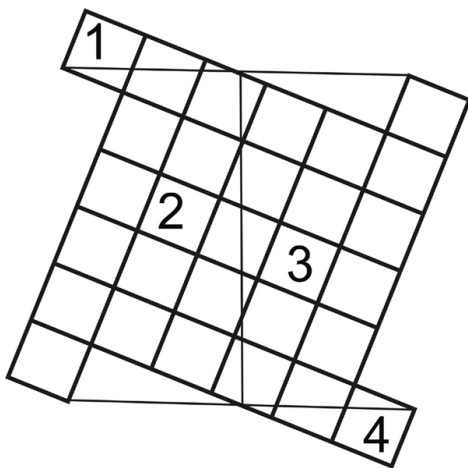
EQUILATERAL DISSECTION



EIGHT PIECES OF CAKE



Make the first two cuts on top of cake in cross fashion and then the third cut from the middle of the cake to have EIGHT equal pieces. *Think Laterally!*

ONE LAST PROBLEM

Cut along the lines and arrange to form a square with $\sqrt{29}$ units as one side.

Chapter 35 WHY WORRY ABOUT POPULATION EXPLOSION?

We are all so concerned about living space, resources for the next generation, environment etc., all on account of growing population. The world isn't big enough to accommodate so many people are oft quoted remarks.

By a certain year there would be no food or space for human beings. We might have to consider living on some other planet or in buildings made in space. A small calculation came to my mind that after all how much space is really required for all the people in the world to stand side by side.

So I just wondered if all the people in the world were told to stand side by side, giving them enough space to stand freely, how much space would they occupy?

If we assume each person to be of normal height and weight, a square of **2 feet by 2 feet** would be sufficient.

Considering population of the world to be 6,000,000,000 i.e. 6 billion or 600 crore, the space required would be

$$(6 \times 10^9) \times (2 \times 2) \text{ sq feet area.}$$

What does this amount to? If we have a square space the length of one side of the square would be

Square root of $\{(6 \times 10^9) \times (2 \times 2)\}$
which is just about **155000 feet.**

In 1 km there are about 3300 feet, so the square size would be just 47 kilometers!

Then where is the problem, the whole world can be adjusted in Delhi itself, with some adjoining space of Gurgaon. *Think again a city 47 km wide and 47 km long can accommodate the whole world.*

Okay what about India how big a city would be required for packing up the Indians together.

There are about 100 crores of us.

On the same lines, the square size required is

Square root of $\{(1 \times 10^9) \times (2 \times 2)\}$,

about **63000** feet only which is just **19 km x 19 km**,

So towns of Ahmedabad or Lucknow are big enough for all Indians to be packed together.

Another bloody question, after how much blood is there in the Universe (*hoping only we live here*). It would also be interesting to find how big a container will be required for storing all the blood.

An average human being has about 4 litres of blood, and there are 6 billion people on earth, i.e the total amount of blood in the universe is

$6,000,000,000 \times 4 = 2.4 \times 10^{10}$ litres.

Now $1\text{m}^3 = 1000$ litres

Thus we would require a cube with volume **$2.4 \times 10^7 \text{ m}^3$**

This means that the side of cube will be about **290 m** , that's all isn't it too small.

If we consider turning the racecourse at Delhi to a container for all the blood with an area of about 2 Sq km , the depth required would be just 12 m . Interesting!

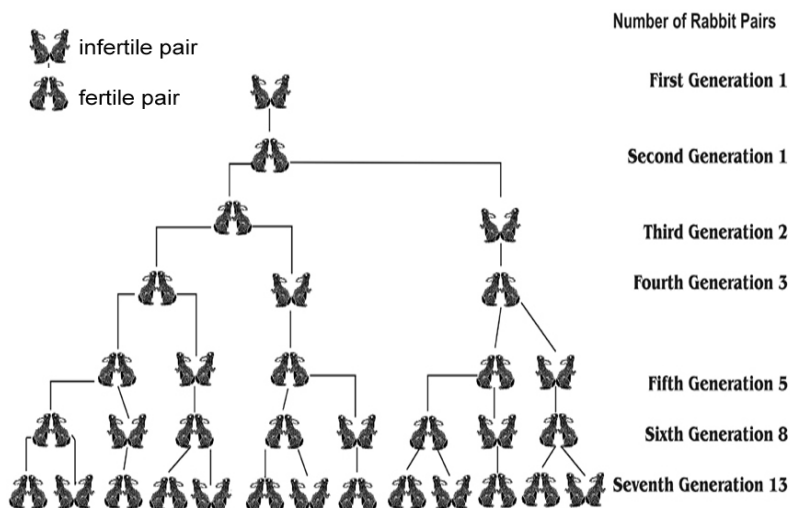
One calculation revealed that speed of growth of human hair is about **$10^{-8} \text{ km per hour}$** . Quick.

Try finding the speed of your growth from less than 18 inches to whatever your height is now!

Chapter 36 NATURE'S OWN SERIES – FIBONACCI

In 1202 the mathematician Leonardo of Pisa, also called Fibonacci, published an influential treatise, *Liber Abaci*. It contained the following recreational problem: “How many pairs of rabbits can be produced from a single pair in one year if it is assumed that every month each pair begets a new pair which from the second month becomes productive?” Straightforward calculation generates the following sequence with following assumptions

- in the first month there is just one newly-born pair,
- new-born pairs become fertile from their second month on
- each month every fertile pair begets a new pair, and
- the rabbits never die



The bee ancestry code

Fibonacci described the sequence "encoded in the ancestry of a male bee." One can derive this series by taking the following facts:

- If an egg is laid by a single female, it hatches a male.
- If, however, the egg is fertilized by a male, it hatches a female.
- Thus, a male bee will always have one parent, and a female bee will have two.

If one traces the ancestry of this male bee (1 bee), he has 1 female parent (1 bee). This female had 2 parents, a male and a female (2 bees). The female had two parents, a male and a female, and the male had one female (3 bees). Those two females each had two parents, and the male had one (5 bees). If one continues this sequence, it gives a perfectly accurate depiction of the Fibonacci sequence.

1,1,2,3,5,8,13,21,34,55,89,144

The series represents the first 12 terms of the sequence now known by Fibonacci's name, in which each term (except the first two) is found by adding the two terms immediately preceding; in general, $F_n = F_{n-1} + F_{n-2}$, a relation that was not recognized until about 1600.

The most astonishing replication of this series in nature is in PHYLLOTAXIS. Phyllotaxis is the distribution or arrangement of leaves on a stem. This term is more than just the distribution of leaves; it extends to petals, seeds,

florets, and branches. If we look at a plant from above, the leaves are not arranged directly above one another, but in a way that optimizes their exposure to sun and rain. The Fibonacci numbers occur when counting both the number of times we go around the stem from one leaf to the next, and when counting the number of leaves we meet until we encounter one directly above the starting one. The number of turns in each direction and the number of leaves are usually three consecutive Fibonacci numbers.

These arrangements are very closely related to the Fibonacci series and The Golden Ratio ϕ , which is another great nature's numbers explained later, **1.618034...**

The interesting thing is that as the series grows and approaches infinity the ratio of the successive terms is ϕ ,

$$\lim_{n \rightarrow \infty} \frac{F_n}{F_{n-1}} = \phi(1.618034...),$$

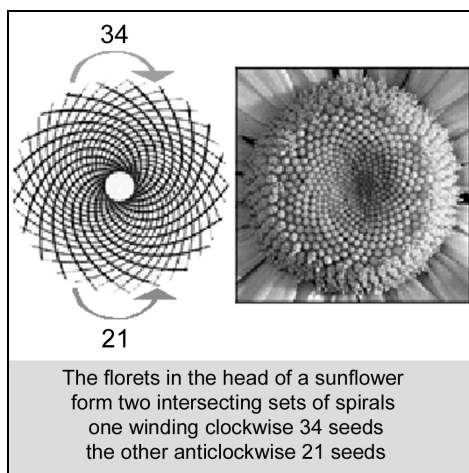
where F_n is the n^{th} term in the Fibonacci series.

Lilies have 3 petals, buttercups 5, delphiniums 8, marigold 13, asters 21 and daisies 34,55 and sometimes 89. The principle that plants follow is the optimal fashion in which space is occupied, receive sunlight and interaction is done with environment.

As the branch grows outwards the leaves are generated at regular angular intervals. But these intervals are not exact rational multiples of 360° . An examination of these generations show that leaves are generated (*approximately*) after $2/5$ of a revolution of a circle for oak, cherry, apple,

plum, $1/2$ for some grasses, lime, linden, $1/3$ for beech, hazel, $3/8$ for rose, polar, willow and $5/13$ for almond. All these approximations are called phyllotactic ratios and *the numerator and denominator are Fibonacci numbers*.

The arrangement of seeds on a sunflower is perhaps the most illustrative example. The seeds are arranged in two families of spirals, 34 winding clockwise and 55 anti clockwise or 55, 89 in same order. In larger sunflowers it maybe 89, 144. The same pattern is observed in daisies. Why does this happens is for the botanist to find out, but all we knew is it was for sunlight seeking.



Many of nature's patterns are related to the golden section and the Fibonacci numbers. For instance, the golden spiral is a logarithmic or equiangular spiral – a type of spiral found in unicellular foraminifera, sunflowers, seashells, animal horns and tusks, beaks and claws, whirlpools, hurricanes, and spiral galaxies. An equiangular spiral does not alter its shape as its size increases. Because of this

remarkable property (known as self-similarity), it was known in earlier times as the ‘miraculous spiral’.

Here is something arithmetically interesting about this sequence,

$$1^2 + 1^2 = 1 \times 2$$

$$1^2 + 1^2 + 2^2 = 2 \times 3$$

$$1^2 + 1^2 + 2^2 + 3^2 = 3 \times 5$$

$$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 5 \times 8$$

$$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 8 \times 13$$

Can you see the pattern?

The sum of squares of the Fibonacci terms is equal to *product of the last term and term next in the series*.

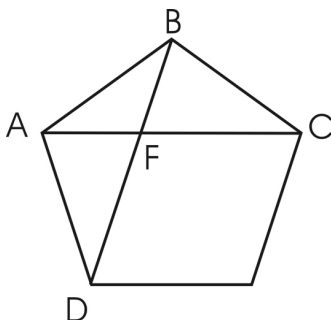
$$\text{i.e. } 1^2 + 1^2 + 2^2 + 3^2 + 5^2 + \dots + F_n^2 = F_n \times F_{n+1}$$

Chapter 37 NATURE'S OWN NUMBER – ϕ (PHI)

Aesthetic judgments are ephemeral and may be developed within the culture of a mathematical age and culture. Their validity is similar to that of a school or period of art. It was once maintained that the most beautiful rectangle has its sides in the **golden ratio**.

The aesthetic delight in the golden ratio, ϕ (Greek Alphabet “phi”) appears nowadays to derive from the diverse and unexpected places in which it arises.

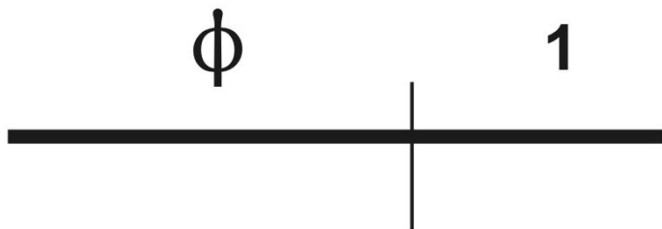
There is, first, the geometry of the regular pentagon.



The diagonal AC divides the diagonal BD in the golden ratio ϕ . The importance of this ratio is, if F is the intersection of the two diagonals then,

$$\frac{BF}{FD} = \frac{FD}{BD}$$

The golden ratio is thought of as the divine ratio, it is the most perfect way of dividing a rectangle.

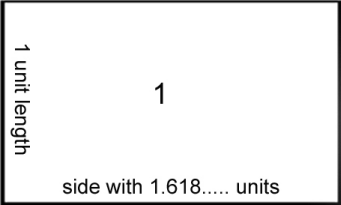
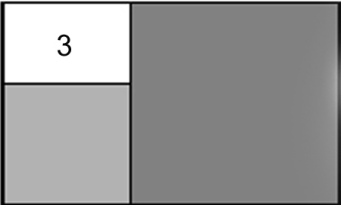


$$\varphi : 1 = (\varphi + 1) : \varphi$$

The line segment is divided at C so that ratio of larger part to smaller part is same as whole to the larger part. The Greeks called it as the method of dividing a line into “extreme and mean ratio”.

Interestingly, if we draw a rectangle whose sides are in the golden ratio, this golden rectangle can be formed out of nested set of ever decreasing golden rectangles. The figure makes this evident. By marking a square of unit 1 from the rectangle, a smaller golden rectangle is formed and process can be continued recursively.

The figure clearly brings out the nested rectangles each having their sides in the golden ratio and are called Golden Rectangle (GR). *It is important to note that this characteristic is only true for the golden rectangle.*

	With one side of 1 unit GR no 1 is constructed
	Taking out unit square GR no 2 is created
	From GR 2, square is taken out to carve GR no3
	From GR no 3 GR no 4 is carved out
	From GR no 4 GR no 5 is made
{GR stands for Golden rectangle}	
Nested Golden Rectangles this is reversible also, starting with a GR add square of longer side, to get another GR and so on. This property is experienced only with GR	

This ratio can be calculated using some algebra as

$$\varphi = \frac{1 + \sqrt{5}}{2} = 1.61803398 \dots$$

What is exciting about this number is that it is equal to one plus its reciprocal.

$$\varphi = 1 + \frac{1}{\varphi}$$

But what really is so special about this number. It is believed (not really very scientifically) that God has created everything with this ratio in mind. You may read Dan Brown's "The Da Vinci Code" for more on this. All the artists of the Renaissance period used this ratio for their art forms like the canvas ratio, human figures etc.

Look at this continued fraction,

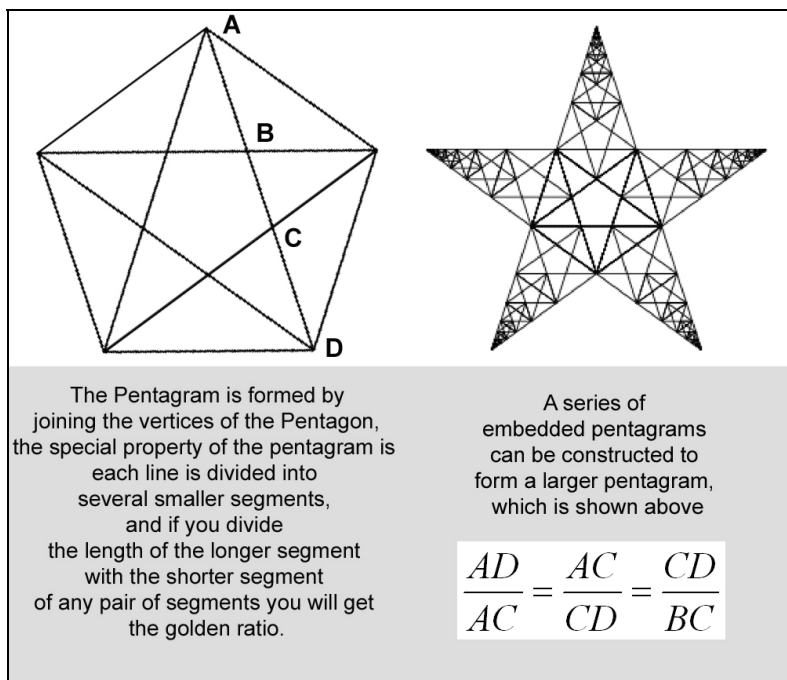
$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \Lambda}}}}$$

Also,

$$\varphi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$$

So certainly there is something special about this ratio. As discussed in the chapter on Fibonacci series, the ratio of successive terms of this series finally converges to ϕ .

The pentagram has long been associated with the planet Venus, and the worship of the goddess Venus, or her equivalent. It is also associated with the Roman Lucifer, who was Venus as the Morning Star, the bringer of light and knowledge. The diagram below depicts the role of golden mean with pentagram.



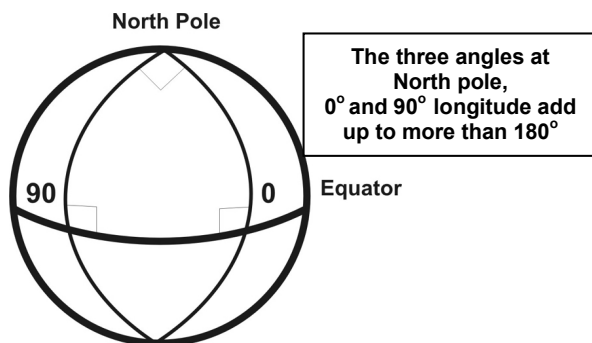
Chapter 38 REDEFINED GEOMETRY IN UNIVERSE

The conventional geometry that we all study and know is called Euclidean Geometry, after the famous Greek Mathematician, Euclid who built the foundations of the subject in his seminal work “ELEMENTS”.

But God had something else in mind which man discovered later and called NON EUCLIDEAN GEOMETRY.

Where on earth can you find a triangle each of whose interior angles is 90° ?

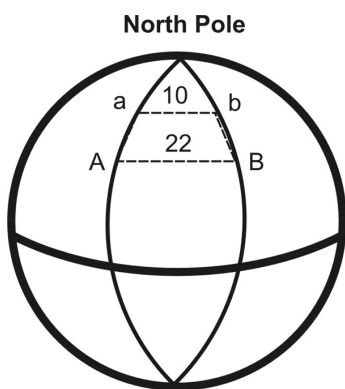
Have you understood the question? Our conventional knowledge tells us that the sum of all interior angles of a triangle is 180° . In this case if all angles were 90° the sum of all angle would be 270°



The lines of longitude corresponding to 0° and 90° and the equator. These lines intersect in a triangle whose angles are all 90° . This is the basic tenet of non Euclidean geometry.

On spherical surfaces the shortest distance between two points are always part of the great circle, so in this sense the great circles play the same role as that of straight lines in a plane. In fact on a sphere, triangles formed from these circles can have sum of all angles anything between 180° and 360° .

Two explorers are 10 km apart when they set off in the same direction and travel at the same speed for two hours. After this they are 22 kms apart.
How is this possible?



The explorers start from **a** and **b**, move towards the equator on different longitudes to reach **A** and **B** which are more distant than **a** and **b**, as depicted in the figure.

If they start 10 kms apart on different longitudes near the North Pole and walk towards south they would soon diverge. There are other ways also.

Well, dear reader its not all, there is just too too much to this. But all that is beyond the scope of this book. Only thing to know is that Einstein successfully predicted the deflection of light, which has no mass, in the vicinity of a star or other massive body. This was an extravagant piece

of geometrizing—the replacement of gravitational force by the curvature of a surface. In relativity theory time is considered to be a dimension along with the three dimensions of space. On the closed four-dimensional world thus formed, the history of the universe stands revealed by the tenets of non-Euclidean universe.

Chapter 39 HAPPY NUMBERS

Happiness is all that we want, even some numbers are intrinsically happy.

A number is called happy if sum of the squares of its digits, and then the sum of the squares of the digits of this sum and so on ends in 1.

For example

$$\mathbf{4599 \Rightarrow 4^2 + 5^2 + 9^2 + 9^2 = 203}$$

$$\mathbf{203 \Rightarrow 2^2 + 0^2 + 3^2 = 13}$$

$$\mathbf{13 \Rightarrow 1^2 + 3^2 = 10}$$

$$\mathbf{10 \Rightarrow 1^2 + 0^2 = 1}$$

Can you try and find out some more happy numbers?

Any number formed with digits 9,8,5,2,1,1 such as 129851 will be happy as

$$\mathbf{9^2 + 8^2 + 5^2 + 2^2 + 1^2 + 1^2 = 176}$$

And 176 itself is a happy number.

It can be appealing to search for more such numbers, as this endeavour leads to some very interesting properties about numbers.

Chapter 40 THE FIVE NINES

$$9 + \frac{99}{99}$$

$$9 + 9 - 9 + \frac{9}{9}$$

$$\left\{9 + \frac{9}{9}\right\} \times \frac{9}{9}$$

$$9 + \frac{9+9}{9+9}$$

$$9 + \frac{9 \times 9}{9 \times 9}$$

$$\sqrt{9} + \sqrt{9} + \sqrt{9} + \frac{9}{9}$$

$$9^{\frac{9}{9}} + \frac{9}{9}$$

Does your mathematical sense trigger? All these result in TEN. *FIVE NINES TO MAKE A TEN*. Find some more like these. Believe me, there are at least four more ways.

Chapter 41 THE GOOD NUMBER 24

24 IN MANY WAYS

24 is an interesting number. It can be made using same digits three times with any mathematical symbol.

$$\mathbf{8+8+8=24}$$

Try with the other numbers.

$$\mathbf{22 + 2 = 24}$$

$$\mathbf{3^3 - 3 = 24}$$

$$\mathbf{4! \times (4 / 4) = 24}$$

$$\mathbf{5! / \sqrt{5 \times 5} = 24}$$

$$\mathbf{(\sqrt{9})! + 9 + 9 = 24}$$

$$\mathbf{(\sqrt{9})^{(\sqrt{9})} - (\sqrt{9}) = 24}$$

There are more such expressions and trying to find them will be worth the effort.

Chapter 42 SUM EQUAL TO PRODUCT, REALLY

There is an interesting property of pair of numbers such that the product of the number is equal to their sum.

For example

3 and 1.5

$3 \times 1.5 = 4.5$

$3 + 1.5 = 4.5$

3.5 and 1.4

$3.5 \times 1.4 = 4.9$

$3.5 + 1.4 = 4.9$

In general if $a = \frac{b}{(b-1)}$ then $a+b=ab$.

Well is it also possible that the sum of three numbers is equal to their product?

Lets see

7.5, 1.66 and 0.8, both the sum and product are 9.96.

Try finding out some more numbers (without reading the next line).

In general, $abc = a + b + c$ if $c = \frac{(a+b)}{(ab-1)}$

Here is a famous 7-11 store problem.

In a "seven-eleven" (7-11) store, a customer selected four items to buy. The cashier at the counter says that he multiplied the costs of the items and obtained *exactly* 7.11, the very name of the store! The customer angered tells him to add and not multiply. The total still was Rs 7.11.

What are the exact costs of the 4 items?

Sol: $3.16 + 1.25 + 1.50 + 1.20$

There are a few more numbers like these,
Check for

$$\mathbf{6.44 = 1.84+1.75+1.60+1.25}$$

$$\mathbf{6.51 = 2.00+1.86+1.40+1.25}$$

Keep trying you will find lots of such numbers.
Mathematicate and meditate.

Chapter 43 TRUNCATING PRIMES

There are a few primes who are so obsessed with remaining primes that even after chopping them off their last digits they remain primes.

Here is an interesting Prime number

593993, this number is a prime. Strangely truncating (*removing the last digit*) this number we find

59399

5939

593

59

5

These are all prime numbers, very fascinating.

The following numbers have the same property.

73939133

23399339

29399999

37337999

59393339

When we speak of primes here is an interesting number sequence.

31
331
3331
33331
333331
3333331
33333331

These are all primes. Does the next number in series also follow the rule, 33,33,33,331?

It has been found that $17 \times 1,96,07,843 = 33,33,33,331$.

Prime numbers are truly the most fascinating aspect of numbers.

Chapter 44 WHY 360° IN A CIRCLE

Surely, sometime you must have wondered why 360 degrees *of all numbers* is one circle, before you simply accepted it as a fact. But the author did not and read some more books to find some facts that may be the reason.

The Babylonians used base 60 notations which is convenient to divide a whole into 2, 3, 4,... 30 parts. Early Greeks then probably divided the radius of a circle into 60 parts. Hence, the diameter had 120 parts. As π was known to be close to 3, the circumference was taken to comprise 360 parts.

Not being a geometry manual, the Bible just picked out a simple approximation to π to convey the order of magnitude of the measured quantity. Nowadays, the same mistake is often committed by users of calculators many of whom can't tell the difference between $1/3$ and 0.3333333333."

Even the Babylonians easily constructed an equilateral triangle. It really makes sense to that this powerful symbol would result in the Babylonians designating one of its angles as **1 base 60 unit**. Whatever maybe the units there are exactly 6 equilateral triangles in a complete circle (*the circle of course circumscribing the triangles*), thus you have 6×60 , 360 degrees in a circle.

Chapter 45 KAPREKAR NUMBERS

9999 is a Kaprekar number. But what is a Kaprekar number.

A Kaprekar number is a special n digit number, such that if it is squared, the sum of the squared quantity's right most n digits and remaining part are equal to the number itself.

Take 9

$$9^2 = 81 \text{ and } 81 = 9+1$$

Similarly

$$703^2 = 494\,209, \text{ thus } 494 + 209 = 703 !!!$$

Try out 500500 .

Shri Dattathreya Ramachandra Kaprekar (1905- 1986) was an Indian mathematician, whose name is associated with a number of concepts in number theory. He was born in Dahanu, near Mumbai, in India.

Even as a small child, his passion was for numbers,

He received his secondary school education in Thana and studied at Fergusson College in Poona. Kaprekar received his bachelor's degree from University of Bombay in 1929. From 1930 until his retirement in 1962, he worked as a schoolteacher in Devlali, India. Kaprekar discovered many

interesting properties in recreational number theory. He published extensively, writing about such topics as recurring decimals, magic squares, and integers with special properties.

There is something very interesting which was discovered by Kaprekar, known as ***Kaprekar's operation***.

First choose a four digit number where the digits are all different (that is not 1111, 2222,...). **Then rearrange the digits to get the largest and smallest numbers these digits can make.** Finally, subtract the smallest number from the largest to get a new number, and carry on repeating the operation for each new number.

Let us try 2006 (the current year)

$$\begin{aligned}6200 - 0026 &= 6174 \\7641 - 1467 &= 6174\end{aligned}$$

6174 repeats itself. Very interesting.

Let us try 2005

$$\begin{aligned}5200 - 0025 &= 5175 \\7551 - 1557 &= 5994 \\9954 - 4599 &= 5355 \\5553 - 3555 &= 1998 \\9981 - 1899 &= 8082 \\8820 - 0288 &= 8532 \\8532 - 2358 &= 6174 \\7641 - 1467 &= 6174\end{aligned}$$

So this number 6174 is really unique and is true for all 4 digit numbers which have different digits. You can try more.

Something that came to my mind and would have definitely rung in the reader's mind, was whether this is true for all numbers consisting of 2 digits, 3 digits etc. Well it is not true for 2 digit numbers but for 3 digit numbers the magic figure is 495.

Try 520

$$520-025=495$$

Try 765

$$765-567=198$$

$$981-189=792$$

$$972-279=693$$

$$963-369=594$$

$$954-459=495$$

again

954-459=495 it repeats itself.

Kaprekar published this article as "Another Solitaire Game", *Scripta Mathematica*, vol 15, pp 244–245 (1949).

Noon is NOT related to the number twelve, but to the number nine. It is derived from the Latin *nona* (ninth) and originally referred to the ninth hour after sunrise, which was closer to the present 3PM.

Chapter 46 THE STORY OF e AND π

*Pi goes on and on and on ...
And e is just as cursed.
I wonder: Which is larger
When their digits are reversed?*

There are two friends in city of maths, named e and π (the Greek letter pi). They have an interesting story to tell, a very strange relationship with a very imaginary citizen called i , who is the square root of -1 . Father Euler got them together in such a way that none could understand,

$$e^{i\pi} + 1 = 0$$

Richard Feynman , the great American Physicist called Euler's formula (from which the identity is derived) ***"the most remarkable formula in mathematics"***. Feynman, as well as many others, found this formula remarkable because it links some very fundamental mathematical constants.

Euler, Leonhard, Swiss mathematician and physicist, one of the founders of pure mathematics was born on April 15, 1707, Basel, Switzerland, died Sept. 18, 1783, St. Petersburg, Russia. He introduced many current notations, such as Σ for the sum; $\sum$ for the sum of divisors of n ; the symbol e for the base of natural logarithms; a , b , and c for the sides of a triangle and A , B , and C for the opposite angles; the letter " f " and parentheses for a function; the use

of the symbol π for the ratio of circumference to diameter in a circle; and i for $\sqrt{-1}$. Thus in his honour the natural logarithm 2.718... was designated as e .

Most of us know π , pi is the ratio of circumference of a circle to its diameter. Many of us may have heard of the natural logarithm e , and wondered why such a strange number for logarithms when base 10 could have worked. The square root of -1 , i is some very impossible kind of number invented by Euler to solve some very complex, commonly incomprehensible equations.

Let us start with π , its value is $22/7$ or 3.14 as commonly known. The history of value of π is stimulating.

Archimedes calculated that π is between $3^{10/71}$ and $3^{10/70}$ ($22/7$), while Chinese scholars around 500 BC showed that π is between 3.14152927 and 3.1415926. In 1596 Ludolph of Cologne used the method of comparing a circle to the straight-sided figure that is approximately a circle, a regular polygon with many sides to calculate π to 32 places. His result was engraved on his tombstone and to this day Germans call π the *Ludolphine number*.

In the 17th century *John Wallis* discovered

$$\pi/2 = 2/1 \times 2/3 \times 4/3 \times 4/5 \times 6/5 \times 6/7 \times \dots$$

in which the numerators are the even numbers from 2 given twice, while the denominators are a similar pattern of odd numbers. *James Gregory* and *Wilhelm Gottfried Leibniz* discovered an even simpler pattern for an infinite sum:

$$\pi/4 = 1/1 - 1/3 + 1/5 - 1/7 + 1/9 - 1/11 + \dots$$

In the 18th century, *Johann Lambert* solved one of the problems connected with π . He showed that π is irrational; in other words, it cannot be expressed as a finite decimal, nor can it have a simple repeating pattern as a decimal.

Nothing stops people from doing the weirdest of things. There are scores of computers and mathematicians calculating the value of π , and there is stiff competition to find the value of the π up to largest decimal place. In 2002 Kanada and fellow researchers at Japan's Information Technology Center set a new record for finding the number of digits of π , a whopping **1.24 trillion** decimal places. This calculation took about *six hundred* hours.

π can be expressed as continued fraction,

$$\frac{4}{\pi} = 1 + \frac{1}{3 + \frac{4}{5 + \frac{9}{7 + \frac{16}{9 + \frac{25}{11 + \frac{36}{13 + \Lambda}}}}}}$$

Notice the odd numbers and squares. Amazing!

$\pi = 3.1415926535897932384626433832795028841971693993751 \dots$

π is such a beautiful number and it can be expressed in various interesting series.

$$\frac{\pi}{2} = 1 + \frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3}{3.5.7} + \dots \text{by Euler}$$

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$$

Using only odd numbers,

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$$

The Euler's number, e is a very natural number and it simply is there. The reason why e is what it is and why it is chosen as a logarithm base is because it is the only logarithm base which has a rate of change the same as the thing which is changing. *If something was getting bigger at a rate proportional to how big it was already it would be termed "exponential growth".*

The function e^x is clearly important, as it is the only function that remains un-altered when differentiated. The number e came to the notice of many mathematicians at about the same time in the early 18th Century. It appeared as a result of several different lines of investigation and became known as a *natural number*.

Yes, the number e does have physical meaning. It occurs naturally in any situation where a quantity increases at a rate proportional to its value, such as a bank account producing interest, or a population increasing as its members reproduce. The number e is the factor by which a bank account earning continually compounding interest (or a reproducing population whose offspring are themselves capable of reproduction, or any similar quantity that grows at a rate proportional to its current value) will increase, if, without the compounding (or without the offspring being capable of further reproduction) it would have doubled (increased by 100%).

Obviously, the quantity will increase more if the increase is based on the total current quantity (including previous increases), than if it is only based on the original quantity (with previous increases not counted). How much more? The number e answers this question.

To put it another way, the number e is related to the how much more money you will earn under *compound interest* than you would under simple interest.

Question: *If you would earn 100% interest (i.e., your money would double) under simple interest, how much money would you end up with under compound interest?*

Answer: *You would have e times your original amount*

$$e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \Lambda \quad \Lambda \quad \Lambda$$

$$e^x = \left(1 + \frac{1}{n}\right)^{nx}, \text{ when } n \text{ tends to infinity.}$$

The principal, which has its interest added to it at regular periods of time is called compounding of interest. It increases by jumps at these interest periods in the following manner. If r is the rate of interest and moreover the interest accrued is added to the principal at the end of each year, then after x years the accumulated amount of an original principal of 1 will be

$$(1 + r)^x$$

if the principal had the interest added to it not at the end of each year, but at the end of each p -th part of a year, then after x years the principal would amount to

$$\left(1 + \frac{r}{p}\right)^{xp}$$

For simplifying let us consider this only for 1 year thus $x=1$ now the interest calculated this way reveals that the principal 1 amounts after one year to

$$\left(1 + \frac{r}{p}\right)^p \text{ let } p/r = n, \text{ thus we have } \left(1 + \frac{1}{n}\right)^{nr}$$

If we now let the interest be calculated at shorter and shorter intervals, i.e. *p tends to infinity or n tends to infinity* the limiting case will signify in a sense that the interest is compounded continuously, at each instant; and we see that the total amount after one year will be e^r times the original principal (remember the formula for e given before). Similarly, if the interest is calculated in this manner, an original principal of 1 will have grown after x years to an amount e^{rx} , where x may be any number, integral or otherwise.

Now the reader would appreciate why this strange number $e = 2.71828182845904523536028747135266249775$ is used as the natural logarithm. One more fact, the differential of e^x is e^x itself !

e , can be expressed as a continued fraction,

$$e = 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5 + \frac{1}{6 + \Lambda}}}}}}$$

What do you say of this $\frac{e^n \cdot n!}{n^n \cdot \sqrt{n}} = \sqrt{2\pi}$

Chapter 47 THE PLATONIC SOLIDS

Regular Solids have been a subject of intrigue and mystery to all. The solids of a special property were assigned to elements of nature. Geometric solids whose faces are all identical, regular polygons meeting at the same three-dimensional angles are called Platonic Solids. There are five such solids, *tetrahedron (or pyramid)*, *cube*, *octahedron*, *dodecahedron*, and *icosahedron*.

The tetrahedron, with its sharp points and edges, was assigned to the element fire; the cube, with its four-square regularity, to earth; and the other solids invented from triangles (the octahedron and the icosahedron) to air and water, respectively. The one remaining regular polyhedra, the dodecahedron, with 12 pentagonal faces, was assigned by Plato assigned to the heavens with its 12 constellations. Because of Plato's development of a theory of the elements of universe based on the five regular polyhedra, they became known as the *Platonic* solids.

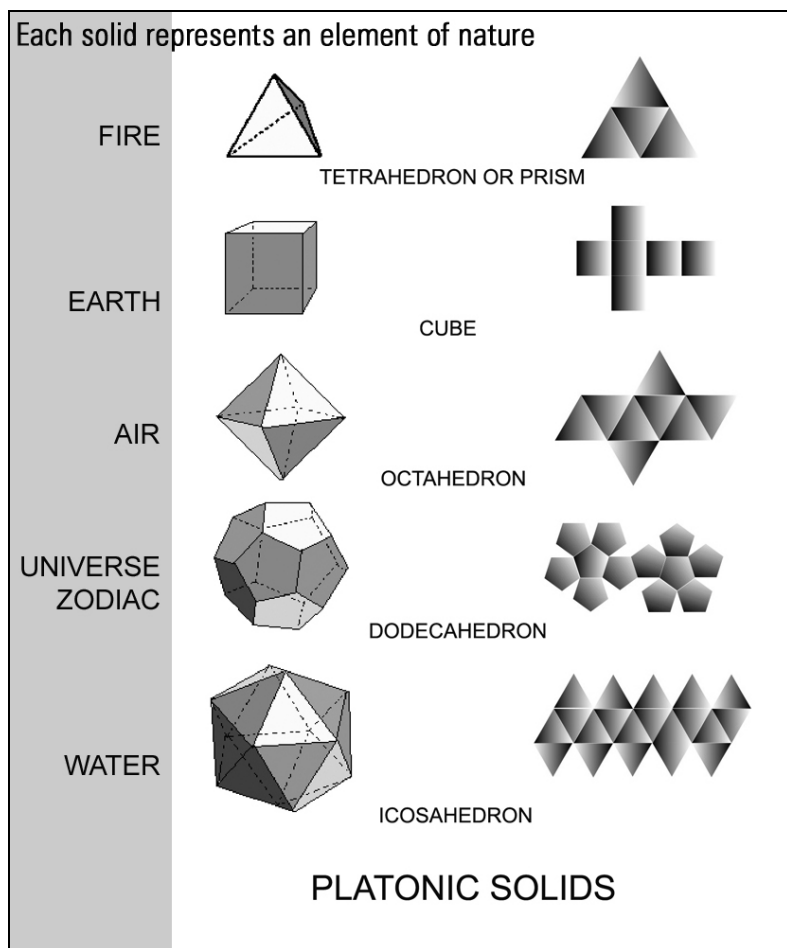
Kepler used the Platonic solids to help explain his model of the cosmos, based on assigning the orbit of each of the known planets to a different, concentric sphere. The Platonic solids (tetrahedron, cube, octahedron, dodecahedron, and icosahedron) just fit between the concentric spherical shells within which, Kepler supposed, the six known planets revolved around the Sun. The cube stands between the shells of Saturn and Jupiter, the pyramid between those of Jupiter and Mars, and so on.

Now what are these solids,

A platonic solid is a polyhedron all of whose faces are congruent regular polygons, and where the same number of faces meet at every vertex. The best know example is a cube (or hexahedron) whose faces are six congruent squares. The basic polygons, triangle, square, pentagon are used for forming the solids. This is how

- ***Triangles.*** The interior angle of an equilateral triangle is 60 degrees.
 - ⇒ 3 triangles meet at each vertex. This gives rise to a *Tetrahedron*.
 - ⇒ 4 triangles meet at each vertex. This gives rise to an *Octahedron*.
 - ⇒ 5 triangles meet at each vertex. This gives rise to an *Icosahedron*.
- ***Squares.*** Since the interior angle of a square is 90 degrees, at most three squares can meet at a vertex. This is indeed possible and it gives rise to a *hexahedron or cube*.
- ***Pentagons.*** As in the case of cubes, the only possibility is that three pentagons meet at a vertex. This gives rise to a *Dodecahedron*.

Hexagons or regular polygons with more than six sides cannot form the faces of a regular polyhedron since their interior angles are at least 120 degrees., and the special condition is not fulfilled. The special condition is that the interior angles of the polygons meeting at a vertex of a polyhedron should ***add to less than 360 degrees.***



All the Platonic solids possess three concentric spheres

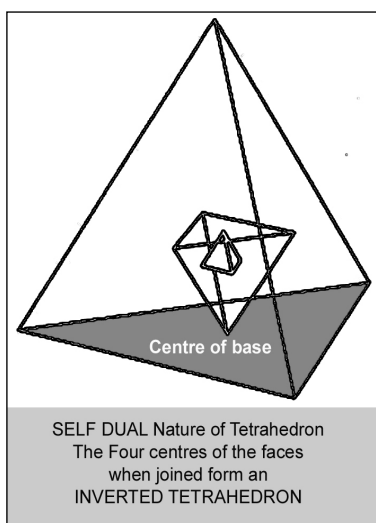
- the circumscribed sphere which passes through all the vertices,
- the midsphere which is tangent to each edge at the midpoint of the edge, and
- the inscribed sphere which is tangent to each face at the center of the face.

The diagram shows on right side the expansion of the solid. The tetrahedron, or pyramid (with 4 triangular faces); the cube (with 6 square faces); the octahedron (with 8 equilateral triangular faces); the dodecahedron (with 12 pentagonal faces); and the icosahedron (with 20 equilateral triangular faces).

These platonic solids are intimately connected with the golden section. Their striking beauty is derived from the symmetries and equalities in their relations. The symmetry of the platonic solids has appealing properties.

For instance, the cube and octahedron both have 12 edges, but the numbers of their faces and vertices are interchanged (cube: 6 faces and 8 vertices; octahedron: 8 faces and 6 vertices). Similarly, the dodecahedron and icosahedron both have 30 edges, but the dodecahedron has 12 faces and 20 vertices, while for the icosahedron it is the other way

round. This allows one solid to be mapped into its dual or reciprocal solid. If we connect the centres of all the faces of a cube, we obtain an octahedron, and if we connect the centres of the faces of an octahedron, we obtain a cube. An icosahedron can be formed similarly from a dodecahedron, and vice versa. The tetrahedron has the **self-dual nature** – *joining the four centres of*



its faces produces another ,inverted tetrahedron.

Interestingly in the Hindu mythology,the icosahedron represents purusha, the male, spiritual principle, and generates the dodecahedron, representing prakriti, the female, material principle.

The properties of the five regular polyhedra are often found in nature's cycles. The platonic solids (especially the tetrahedron, octahedron, and cube) form the basis for the orderly arrangement of atoms in crystals, though the *regular* dodecahedron and icosahedron are never found. Tetrahedral geometry commonly occurs in organic and inorganic chemistry and in submicroscopic structures. For example, the methane molecule (CH_4) is a tetrahedron, with a carbon atom at its centre and a hydrogen atom at each of its four corners.

Carbon exists in three pure forms. In graphite crystals, the carbon atoms lie in hexagonal sheets, which readily slide off a pencil as we write. In diamond, the hardest substance known, each carbon atom is bonded to four others in a tetrahedral arrangement. Buckminsterfullerene, the third, highly stable allotrope of carbon, consists of 60 carbon atoms, arranged at the vertices of a truncated icosahedron (i.e. one with its corners cut off). The great majority of viruses are icosahedral, including the polio virus and the 200 kinds of viruses responsible for the common cold. Icosahedral symmetry is believed to allow for the lowest-energy configuration of particles interacting on the surface of a sphere. The five platonic solids are also found in radiolarian skeletons.

Chapter 48 *i*, IS IT REALLY IMAGINARY?

Despite their name, imaginary numbers are just as "real" as real numbers. One way to understand this is by realizing that numbers themselves are abstractions, and the abstractions can be valid even when they are not recognized in a given context. For example, fractions such as $3/4$ and $5/7$ are meaningless to a person counting eggs, but essential to a person comparing the sizes of different collections of stones. Similarly, negative numbers such as -3 and -5 are meaningless when keeping score in a cricket match, but essential when keeping track of debits and credits in a bank account.

Imaginary numbers follow a similar pattern. For most real tasks, real numbers (or even rational numbers) offer an adequate description of data, and imaginary numbers have no meaning; however, in many areas of science and mathematics, imaginary numbers (and complex numbers in general) are essential for describing reality. Imaginary numbers have essential concrete applications in a variety of sciences and related areas such as signal processing, control theory, electromagnetism, quantum mechanics, and cartography.

For example, in electrical engineering, when analyzing AC circuitry, the values for the electrical voltage (and current) are expressed as imaginary or complex numbers known as phasors. These are real voltages that can cause damage/harm to either humans or equipment even if their values contain no "real part".

The Euler's formula is used widely to express signals (e.g., electromagnetic) that vary periodically over time as a combination of sine and cosine functions. Euler's formula states that, for any real number x ,

$$e^{ix} = \cos x + i \sin x$$

An important Mathematical Warning

The imaginary unit is written $\sqrt{-1}$ in all mathematics contexts, but great care needs to be taken when formulae involving radicals. Quite simply, it can easily be proven that $1 = -1$ (mathematically impossible). Observe,

$$-1 = i \cdot i$$

$$i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{-1 \cdot -1} = \sqrt{1} = 1$$

But what is wrong with this. The calculation rule $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$ is only valid *for real, non-negative values of a and b* .

The word *imaginary* was first applied to the square root of a negative number by Rene Descartes around 1635. Descartes wrote that although one can imagine every Nth degree equation had N roots, there were no real numbers for some of these imagined roots. Around 1685 the English mathematician, John Wallis wrote, "We have before had occasion to make mention of Negative Squares, and Imaginary Roots". Some mathematicians have suggested the name be changed to avoid the stigma that it seems to create in young students; "Why do we have to learn them if

they aren't even real?" Before this misleading word "imaginary", the square roots of negatives had been called *sophisticated* or *subtle*.

i for the imaginary unit was first used by Leonhard Euler (1707-1783) in a memoir presented in 1777 but not published until 1794 in his "*Institutionum calculi integralis*."

Chapter 49 BINARY IS OKAY BUT WHY HEXADECIMAL IN COMPUTERS?

0 and 1 are perhaps the most familiar figures for all computer literates and semi literates. It is also understood that they depict the off and on condition respectively. Those of us who venture just a bit more would have encountered the hexadecimal stuff, base 16 numbers. But why such a strange and complex number which uses 0 to 9 and then the first six alphabets A B C D E F.

Letters are used for the numbers from ten through fifteen

$$A = 10; B = 11; C = 12; D = 13; E = 14; F = 15$$

They can be either upper case or lower case.

The numbers we added above if written as base 16 numbers look like:

$$\begin{array}{r} 3F2 \\ + B37 \\ \hline F29 \end{array}$$

To interpret such numbers as base 10 numbers, one needs to know the powers of 16.

$$\begin{aligned} 16^3 &= 4096 \\ 16^2 &= 256 \\ 16^1 &= 16 \\ 16^0 &= 1 \end{aligned}$$

So the number F29 in base 16 is equal to

$(F \times 16^2) + (2 \times 16^1) + (9 \times 16^0)$ which is calculated as,

$$\begin{array}{r}
 15 \times 256 = 3840 \\
 + \quad 2 \times 16 = 32 \\
 + \quad 9 \times 1 = 9 \\
 \hline
 3881 \\
 \text{in base 10}
 \end{array}$$

Difficult indeed, but a simple property depicted below makes it so easy to operate with. It's so easy to change from base 16 to base 2 and back.

Every hexadecimal digit is broken down into a 4 digit binary number. These digits are just written down in the same order as the hexadecimal number and you've got the equivalent binary (base 2) number.

Lets consider the same example for number F29

F is equal to 15 which is $8 + 4 + 2 + 1$. (These are the powers of 2).

That means that $F = (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

So $F_{16} = 1111_2$ (*The subscripts show the base.*)

Since 2_{16} is (1×2^1) , then $2_{16} = 0010_2$

Since 9_{16} is $(1 \times 2^3 (8)) + (1 \times 2^0)$, then $9_{16} = 1001_2$

Putting this together makes $F29_{16} = 1111\ 0010\ 1001_2$.

To switch between base 2 and base 16 all one only needs to learn writing the numbers 0 to 15 in base 2. When expressing large numbers, the binary system is very inefficient, as is any low base system. Higher base systems can express numbers using fewer characters. To illustrate this, we will look at a table showing numbers, as they would look in different base systems. The columns are different systems: Base 2, 3, 4, 8, 9, 10, 16, and 18. The rows are different numbers as they look in each system.

BASE							
2	3	4	8	9	10	16	18
10	2	2	2	2	2	2	2
1,000	22	20	10	8	8	8	8
11,011	1,000	123	33	30	27	1B	19
1,100,100	10,201	1,210	144	121	100	64	5A
100,000,000	100,111	10,000	400	314	256	100	E4
1,000,000,000	200,222	20,000	1000	628	512	200	1A8

Note: The commas are present only to simplify reading

Notice that it is much simpler to express numbers using larger base systems. Because computers rely on the binary system, base 16 is used for expressing long binary numbers. There is a reason base 16 is used for binary instead of say 10, or 18. *If you look at the table, 100,000,000 binary is the same as 100 hexadecimal, whereas it is '256' in base 10 and 'E4' in base 18.* Hexadecimal is very compatible with binary because 16 is a power of 2. Likewise, base 4 and 8 are also very nice systems to represent binary numbers. This is not only applicable with powers of 2 though. All of the base 3

numbers in the table work well with base 9 (though the only number showing this obviously in the table is '1000'). One thing to note is that base 10 is not "nice" with respect to base 2, and base 18 is not "nice" with respect to base 3, though each is a multiple of the previous. *In order for number systems to "look good together", the higher system must be a power of the lower.*

What does this all mean?

Conversion between binary and hexadecimal numbers is easy, simply group the binary digits in groups of four, starting from the least significant bit, and translate into hexadecimal (inserting leading zeroes in front if we feel the need) , the following examples will clarify.

Base 10	Base 2	Base 16
1	0001	1
10	1010	A
100	0110 0100	64
1,000	0011 1110 1000	3E8
10,000	0010 0111 0001 0000	2710
100,000	0001 1000 0110 1010 0000	186A0
1,000,000	1111 0100 0010 0100 0000	F4240

The base 16 numbering system is used as a shorthand for representing binary numbers. Each half byte (four bits) is assigned a hex digit. Hex values are identified with an "h" or dollar sign, thus \$3E0, 3E0h and 3E0H all stand for the hex number 3E0.

The hexadecimal system is useful because it can represent every byte (8 bits) as two consecutive hexadecimal digits. It is easier for humans to read hexadecimal numbers than binary numbers. It is often termed as HEX. Because it is easy to convert a value from hexadecimal to binary, by merely translating each hexadecimal digit into its 4-bit **binary** equivalent. Hexadecimal numbers have either an **0x** prefix or an **h** suffix.

Binary is an effective number system for computers because it is easy to implement with digital electronics. It is inefficient for humans to use binary, however, because it requires so many digits to represent a number. The number 76, for example, takes only two digits to write in decimal, yet takes seven digits to write in binary (1001100). To overcome this limitation, the *hexadecimal number system* was developed. Hexadecimal is more compact than binary but is still based on the digital nature of computers.

End Note WHAT IS MATHEMATICS?

Here is a little story

An engineer, a physicist and a mathematician are all put in a room with a burning fire in the middle and one bucket of water next to it. This is how they respond:

The engineer, practical as he is, takes the bucket and throws the water over the fire to put it off instantly.

The physicist, curious as he is, takes the bucket and pours out water all around the fire and watches the fire to die slowly.

The mathematician walks about in the room, observes the fire and the bucket of water, thinks for a moment, determines there is a solution and leaves the room again.

Does it explain all? An engineer thinks that his equations are an approximation to reality. A physicist thinks reality is an approximation to his equations. A mathematician doesn't care.

The word "mathematics" comes from the Greek μάθημα (máthema) meaning "science, knowledge, or learning" and μαθηματικός (mathematikós) meaning "fond of learning". It is often abbreviated maths in Commonwealth English and math in North American English. India uses maths. It is

defined as study of quantity, structure, space and change. Maths is used in every society for the purpose of accounting, measuring land, predicting astronomical events. The pure mathematician does not even bother about the practical application of his theory, but the remarkable thing is that it ends up in some very useful application. This prompted Eugene Wigner to lecture on "**the unreasonable effectiveness of mathematics**". Natural sciences depend heavily on new mathematical discoveries.

The elegance of mathematics is intrinsic with its simplicity and generality. There is beauty in a difficult proof, and *if the result is not beautiful it is not thought of as correct*. In *A Mathematician's Apology*, G. H. Hardy has expressed the belief that the aesthetic considerations in themselves are sufficient to justify the study of pure mathematics.

Writings in mathematics are not easily accessible to the layperson. This was the compromise Stephen Hawking made while writing the bestseller *A Brief History of Time*, his book had only one mathematical equation. The mathematical notation is a very recent development (somewhere in the book there is a chapter on this). Before that mathematics was written out in words. In fact now a few symbols contain plenty of information. Common words like "and", "or", "only" have very precise meanings.

A debate, quite unnecessary as I feel, is undertaken on qualification of mathematics as Science. Gauss calls mathematics the Queen of Sciences. Similar to all other fields of science mathematics relies on intuition and experimentation. Since many of the conjectures are formed

our of experimentation, Mathematics can be labeled as a science.

Major disciplines within mathematics arose out of the inevitability for calculations in commerce, understanding relationship between numbers and most importantly predicting astronomical events for the purposes of astrology. This perhaps is the only thing good of astrology that its believers forced development of mathematics. Thus four main fields **arithmetic**, **algebra**, **geometry** and **analysis(calculus)** matured simultaneously complementing each other.

Arithmetic concerns itself with study of quantity, starts with numbers, arithmetical operators. The higher properties are studied in number theory. Algebra is plainly a study of structures. It led to abstract numbers such as square root of 2. Concept of vectors, groups, rings, fields etc. are all developments in algebra. The study of space originates with geometry. It has offshoots like trigonometry, analytic geometry, topology, differential geometry, combining concepts of quantity and space. Calculus was developed as a tool for understanding and describing change. The relationship of quantity and rate of change is differential calculus. An important field in applied mathematics is statistics whose basis is the probability theory and its greatest tool. Numerical analysis takes a broad range of mathematical problems and simplifies them for the computers to solve which otherwise were beyond human capacity. One more story,

The mathematician Mr Math, had to move to a new place. His wife didn't trust him very much, so when

they stood down on the street with all their things, she asked him to watch their ten trunks, while she get a taxi. Some minutes later she returned. Said the husband "I thought you said there were ten trunks, but I've only counted to nine." The wife said: "No, they're TEN!"

"But I have counted them: 0, 1, 2, ..."

Essentially mathematics is nothing more than the language of science. While science is a systematic study of nature, mathematics is a concise form of communication used to represent nature.

There are facets of mathematics, which are bewildering and seem quite unnatural, they seem more like forcefully finding patterns when there is seemingly no symmetry. But that's the fun of abstraction, the license to simply *generalize*.

$$10^2 + 11^2 + 12^2 = 13^2 + 14^2$$

Both the sides add up to 365, numbers of days in a year.
This was the message Russians wanted to send
for the living beings in outer space.

Books to Read

1. Amir Aczel, *Fermat's Last Theorem*, Dell Books, 1996. A history and mathematical discussion of "Fermat's Last Theorem" as studied over the centuries, culminating in its solution by Andrew Wiles.
2. David Blatner, *The Joy of Pi*, Walker Publishing, 1997. This is a very nice account of the history and culture of pi.
3. Calvin Clawson, *Mathematical Mysteries: The Beauty and Magic of Numbers*, Plenum Publishing, 1996. A very well-written lay introduction to number theory.
4. Carl Boyer, *A History of Mathematics*, John Wiley Publishing, 1968. This is one of the classic textbooks on the history of mathematics.
5. John Casti, *Five Golden Rules: Great Theories of 20th-Mathematics - and Why They Matter*, John Wiley Publishing, 1996. This gives an intelligent and readable discussion of some great mathematical ideas of this century, with applications of those ideas outside of mathematics.
6. John Conway and Richard Guy, *The Book of Numbers*, Springer-Verlag, 1996. An interesting presentation and discussion of numbers, from integers to the hyper-reals.
7. Richard Courant and Herbert Robbins; revised by Ian Stewart, *What is Mathematics?*, Oxford University Press, 1996. This is an update of a classic introduction to mathematics.
8. Philip Davis, *The Thread: A Mathematical Yarn*, Harvester Press, 1983. This is a unique and eclectic tale tying together mathematical ideas and personalities across the centuries. An interesting and enjoyable read!
9. Philip Davis and Reuben Hersh, *The Mathematical Experience*, Birkhäuser Publishing, 1981. This a very readable account, incorporating discussions of the major areas of mathematics and the history of mathematics.
10. Heinrich Dörrie, *100 Great Problems of Elementary Mathematics: Their History and Solution*, Dover Publications, 1965. This is an English language translation of the original German text, published in 1958. A book of very interesting solutions, a must for maths lover.
11. C. H. Edwards, Jr., *The Historical Development of the Calculus*, Springer-Verlag, 1979.

12. Clifton Fadiman, *Fantasia Mathematica*, Springer-Verlag, 1997. Reprint from the 1958 publication. Collection of miscellaneous stories and other materials involving mathematics.
13. James Gleick, *Chaos: Making a New Science*, Penguin Books, 1987. This was a very well-read popular account of chaos theory and the associated topics of dynamical systems and fractal geometry.
14. G. H. Hardy, *A Mathematician's Apology* (with a forward by C. P. Snow), Cambridge Univ. Press, Cambridge, 1940. This is a personal (and very famous) accounting of doing mathematics by a 'mathematical great'.
15. Paul Hoffman, *The Man Who Loved Only Numbers*, Fourth Estate Limited, London, 1999. This is a biography of Paul Erdős, a great mathematician of the 20th Century.
16. Andrew Hodges, *Alan Turing: The Enigma*, Simon & Schuster, 1983. This is a biography of one of the most important deep thinkers in the development of modern computing, a man whose name is given to the premier award in computer science, **The Turing Award**.
17. George Gheverghese Joseph, *The Crest of the Peacock: Non-European Roots of Mathematics*. This is an excellent source of history on the contributions to the development of mathematics from ancient Egypt, Babylonia, India, China, the Arab world, and other parts of the non-European world.
18. T. W. Körner, *The Pleasures of Counting*, Cambridge University Press, 1996. A well-written and interesting introductory discussion of many areas of mathematics.
19. George Lakoff and Rafael Núñez, *Where Mathematics Comes From: How The Embodied Mind Brings Mathematics Into Being*, Basic Books, New York, 2000. This is a cognitive analysis of the structure of mathematical ideas.
20. Eli Maor, *e: The Story of a Number*, Princeton University Press, 1994. This is a history of the number e , discussing also the history of π , i , and other important quantities in mathematics.
21. Tristan Needham, *Visual Complex Analysis*, Clarendon Press, 1997. This is a lovely presentation of complex analysis, emphasizing its geometric connections.
22. Ivars Peterson, *The Mathematical Tourist: Snapshots of Modern Mathematics*, Freeman Publishers, 1988.

23. Charles Seife, *Zero: The Biography of a Dangerous Idea*, Penguin Books, New York, 2000. This is an historical and mathematical analysis of the development of the idea of zero in our number system, explaining its historical significance outside of mathematics, especially in religion and philosophy.
24. George Simmons, *Calculus Gems: Brief Lives and Memorable Mathematics*, McGraw-Hill Publishing, 1992. An accounting of the major figures in the development of the calculus, together with a discussion of some of the important mathematical problems studied in this development.
25. David Smith, *A Source Book in Mathematics*, Dover Publications, 1959. This was originally published in 1929, and it contains original writings from important papers of well-known mathematicians, from 1478 onwards.
26. Ian Stewart, *The Problems of Mathematics*, Oxford Press, 1987. Discusses the "nature of mathematics" by looking at particular important problems, many of interest in current applications to real world problems (e.g. cryptology). Stewart is one of the best known popularizers of mathematics.
27. Ian Stewart, *Does God Play Dice? The Mathematics of Chaos*, Blackwell Publishers, 1989. A very readable account of chaos theory and the associated topics of dynamical systems and fractal geometry.
28. Ian Stewart, *Nature's Numbers: The Unreal Reality of Mathematics*, Basic Books, 1995.
29. John Stillwell, *Mathematics and Its History*, Springer-Verlag, 1989.
30. Stewart, Ian. *Game, Set, and Math: Enigmas and Conundrums* Cambridge, MA: Basil Blackwell, 1989.
31. Dudeney, Henry E. *Amusements in Mathematics* Mineola, NY: Dover, 1958.
32. Dudeney, Henry E. *The Canterbury Puzzles* Mineola, NY: Dover, 1958.
33. Dudeney, Henry E. *536 Puzzles and Curious Problems* New York, NY: Charles Scribner's, 1967.
34. Gardner, Martin, ed. *The Mathematical Puzzles of Sam Lloyd*, Mineola, NY: Dover, 1959. 2 Vols.

Mathematics on Internet

A few of the most interesting websites for general as well as recreational mathematics are listed below. They are listed in order of author's preference.

1. mathworld.wolfram.com
2. www-history.mcs.st-andrews.ac.uk
3. www.cut-the-knot.org
4. mathforum.org/library
5. archives.math.utk.edu/topics
6. www.math.com
7. richardphillips.org.uk
8. www.archimedes-lab.org
9. www.worldofnumbers.com
10. www-groups.dcs.st-and.ac.uk
11. www.sunsite.ubc.ca/LivingMathematics
12. www.maa.org
13. hlavolamy.szm.sk
14. www.brainbashers.com
15. www.math.utah.edu
16. thesaurus.maths.org
17. www.lboro.ac.uk
18. www.brainteaser-world.com
19. www.math-atlas.org

20. numbers.computation.free.fr
21. aleph0.clarku.edu
22. www.mathsisfun.com
23. www.perplexus.info
24. www.stetson.edu
25. sciencegems.com/math.html#1
26. plus.maths.org
27. www.heartofmath.com

The Google search can reveal millions of pages on Mathematics. The online encyclopedia Wikipedia, (*en.wikipedia.org/wiki*) has pages of interesting information and links to some of the best websites on almost any topic in Mathematics.